DISCUSSION PAPERS

DP/88/2012

The Predictive Power of Some Market Liquidity Risk Measures: An Empirical Approach

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SUMMARY: The recent global financial crisis and Euroarea Sovereign Debt Crisis have both been characterized by liquidity failures of certain markets which have reinvigorated the discussions about the importance of measurement and management of liquidity risk. Following these discussions, we also focus on market liquidity (trading) risk – i.e. the risk of trading into illiquid market – in the attempt to quantify and apply practically the formalism of liquidity policy, as described by Acerbi and Scandolo*.

We propose a comprehensive approach to market liquidity risk through modeling versatility (shiftability) of assets – the cost for shifting them to cash equivalent – that allows setting of an explicit liquidity risk tolerance levels. The new liquidity measures developed - namely systemic and idiosyncratic illiquidity – do indicate ensuing elevated levels of market volatility and possible market failure. The algorithm utilizes Market Microstructure approach and assesses Marginal Supply-Demand Curves through effective bid-ask spreads. It is applied in LARC software, which demonstrates good predictive power when studying the Euro-Denominated Bond Market in the period October 2008 – November 2009. Thus, the model can also serve as an early-warning system.

The case studies prove that elevated levels of liquidity risk are manifest before the realization of periods of high market volatility and increased perception of credit risk by the market. Its findings confirm that realization of liquidity risk precedes realization of market and credit risk, which vouches for the soundness of the algorithm when assessing issuers' liquidity profile.

We posit that the LARC model – using both its early-warning and optimization capabilities – can be applied to address a number of real-world issues faced by institutional investors and other financial institutions. The selection of a liquid investment universe is one application; another is liquidity optimization that complements traditional Markowitz optimization and/or VaR analysis. Our framework is flexible and modular enough that it can be integrated with or interwoven into existing credit risk systems, depending on the robustness desired and the liquidity policy of the institution.

At the time of the preparation of the paper Tsvetan Manchev was an Associate Professor at "St. Kliment Ohridski" Sofia University, Daniel Simeonov – a risk analyst at the Bulgarian National Bank, Hristo Ivanov was a programmer at Codix, and Christian Hausmann – an analyst in Fixed Income Sales at Deutsche Bank.

The opinions presented in the paper are those of the authors and should not be associated with the institutions in which they worked.

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 $^{^{\}star}$ Acerbi, Carlo and Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk", 2007.

Introduction

As Acerbi and Scandolo (2007) suggest, liquidity risk may foreshadow severe market crises. According to them, "the value of portfolios will no more be necessarily supposed to be a linear map in the vector space of portfolios"¹ thus evoking the need for the establishment of Marginal Supply-Demand Curves (MSDCs) and definition of liquidity policy to manage the manifestations of liquidity risk. In their classification, the liquidity risk is multifaceted and is divided into the following:

Facet 1 – The risk that our portfolio may run short of euros;

Facet 2 - The risk we run trading in an illiquid market;

Facet 3 – The risk of drainage of the liquidity circulating in our economy.²

Thus the definition, measurement, and management of liquidity risk is paramount to developing early warning systems for market dysfunction and proper adjustment of market and credit risk tolerance to the prevailing market conditions. The current paper focuses on a practical approach of tackling Facet 2, i.e. the versatility (shiftability, see BOX 3: Regulatory overview of liquidity risk measurement and management below) of a given asset to maturity of 0 (cash equivalent), and proposes an algorithm of determining MSDCs and quantification of the illiquidity of the market. It develops new liquidity measures - namely systemic and idiosyncratic illiquidity, which indicate ensuing elevated levels of market volatility and possible market failure. The algorithm is applied in the Liquidity Assessment and Risk Calculator (LARC³), which is authors' software solution offering a way of measuring the cost of liquidating portfolio positions in different market conditions. The algorithm underlying LARC uses a Market Microstructure approach (see BOX 2: Market Microstructure concepts applied below), constructing effective bid-offer spreads and taking into account order flow effects.

The paper is organized in two sections followed by concluding remarks and three appendix sections or Boxes giving detailed presentations of the underlying theory.

In Section 1, Market Liquidity Risk Framework and an Overview of the LARC Model, we formally define liquidity risk, point out some specific observations made by Acerbi & Scandolo (2007), and state the problems this paper addresses. Then we propose two measures – Systemic and Idiosyncratic

¹ Ibid., p. 4

² Ibid., p. 2

³ "LARC – Liquidity Assessment and Risk Calculator" is a registered trademark of DCH LLC in the EU and USA, and is registered with the US Copyright Office

Illiquidity and show how they are used in defining liquidity risk tolerance and the assumptions underlying their usage in the LARC model for liquidity policy. We also briefly discuss the concrete software implementation of LARC, which is used in the case studies presented in Section 2.

In Section 2, Case Studies, we consider the viability of the model tested with MTS⁴ data for real trades in the period Jan. 2007 – Nov. 2009. The algorithm is employed to study the Euro-Denominated Bond Market, but may be used to study other markets as well. The whole period Jan 2007 – Nov. 2009 is divided into three sub-periods, which establish separate case studies to test liquidity conditions in the:

- 1. Pre-Crisis (Jan. 2007 June 2007), where we look into structured bonds;
- 2. Aftermath of Lehman Brothers' Failure (Oct. 2008 Aug. 2009) where we consider the cases of Ireland and KfW;
- 3. Run-up to the Eurozone Sovereign Debt Crisis (Sept. 2009 Nov. 2009) where we consider the properties of Greek government bonds that show signs of distress well before their yield spread and CDS levels increased exorbitantly.

In the concluding remarks we sum up the findings and the adequacy of the LARC model as tested against the data and suggest areas for future development and application.

In BOX 1: Definitions of Concepts Underlying the Model we formally define the key concepts related to liquidity risk such as Marginal Supply-Demand Curve (MSDC), Proceeds, Bid-Ask Spread, Liquidation, Liquidity Policy, Uppermost and Mid Mark-to-Market Policies, Order Flow, Liquidity Supplier/Demander, Effective Bid-Ask Spread, and Endogenous and Exogenous Illiquidity. In BOX 2: Market Microstructure Concepts Applied we look more closely at Market Microstructure effects on liquidity as integrated into a comprehensive application of liquidity risk theory, allowing the explicit definition of a Market Liquidity Policy defined in Section 1. In BOX 3: Regulatory Overview of Liquidity Risk Measurement and Management, we discuss the different approaches to managing liquidity risk. and briefly touch on the Regulatory Framework of the Bank for International Settlements (BIS) and the Committee of European Banking Supervisors (CEBS), which both address liquidity risk and put more emphasis on its sound management in the new Basel III framework.

⁴ For more information, see the homepage of MTS Group, http://www. mtsmarkets.com/

Section 1: Market Liquidity Risk Framework and an Overview of the LARC Model

Market Liquidity Risk Framework

Market liquidity risk is the risk of trading in illiquid markets (Facet 2⁵ of liquidity risk presented above) – markets in which higher premium is demanded by the liquidity suppliers for the possibility of trading against informed market participants.

Market Liquidity has at least three dimensions⁶:

Market tightness: "small" buy and sell orders execute at prices that are not too different; can be measured by the quoted spread, i.e. the difference between the best ask price and best bid price for a security.

Market depth: traders can trade large quantities without moving prices much; can be measured by the effective spread.

Market resiliency: the speed at which transient price deviations trig gered by order arrivals dissipate.

Lack of market liquidity is manifested through the difference between the expected fair value proceeds and the the ones received after concluding a transaction and deducting the transaction cost resulting from the bid-ask⁷ spread. This is is a concrete realization of liquidity risk. We essentially run the risk that the proceeds received will not meet our cash outflows and that these proceeds will significantly deviate from the ones suggested by our mark-to-market policy. Thus shifting the assets to maturity 0, i.e. cash equivalent, could potentially cost us more, and there are numerous factors, most stemming from the current market microstructure, which affect the severity of this risk. Therefore, to measure market liquidity we shall construct a model that captures the dynamic of the bid-offer spread, or the effective bid-offer spread.

The principal determinants of effective bid-offer spread (in price terms) on the bond market are assumed to be two factors –size (*Lot Size*) and maturity (*Lot Maturity*). We shall first consider the effect of the size factor, and then look at them jointly. The algorithm uses OLS regression of high-frequency data to construct the dynamic and proposes concrete determinants of en-

 $^{^5}$ Acerbi, Carlo and Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk," 2007, p. 2

⁶ Foucault, Thierry (HEC, Paris), "Market Microstructure Approach", 2007, p. 32

⁷ In this paper, bid-ask and bid-offer spread are used interchangeably, as the ask price is also commonly referred to as offer price in market convention.

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dogenous and exogenous illiquidity⁸ of a security, which are aggregated to issuer level and referred to as idiosyncratic and systemic illiquidity. Hence an explicit method is tested against the MTS Data for expressing liquidity risk tolerance and the eligibility of different issuers form different asset classes for different levels of liquidity risk tolerance.

The results confirm that elevated levels of liquidity risk herald increased volatility and credit concerns and that the current algorithm does provide an early-warning mechanism and sifts affected issuers effectively well in advance.

Below we list the two key observations made by Acerbi and Scandolo, upon which we build the model underlying LARC:

Key Observation 1. In the presence of liquidity risk, the value of portfolios will no more be necessarily supposed to be a linear map in the vector space of portfolios.⁹

As the price of an asset (Definition 1, *Box 1*) determines its value, and a price can be assigned only in the context of the portfolio where the asset is and the conditions in which we decide to realize a trade, assets do not have abstractly defined inherent value. Instead of trying to determine the value as an idea, we have to look at the shadows of its realizations cast in the cave of financial markets, if we put the argument more philosophically in Plato's terms. Hence, liquidity risk, determined by the state of the markets the asset is traded in, as well as a number of exogenous factors such as the availability of this asset or regulations concerning it, lead to different values of the same asset, depending on a number of parameters.

Key Observation 1.2. In the presence of liquidity risk, we cannot speak of the "value" or of the "liquidity risk" of a portfolio unless we fix a liquidity policy.¹¹

Overall, to establish the value of an asset, we need to account for liquidity risk as it affects the price realization that we observe. In order to manage properly this risk, we need to define the liquidity policy of the portfolio (see *Box 1*).

The liquidity policy of a portfolio is dictated by its cash flow profile and the function this portfolio serves for its holder. A portfolio's liquidity gaps are

⁸ zeb/rolfes.schierenbeck.associates, Mag. Peter Madritsch, "Liquidity Risk – Alternatives for Quantification of Liquidity Risk", p. 19, pres. at Deutsche Bundesbank and Oesterreichische Nationalbank Advanced Seminar on Basel II, 2009

 $^{^9}$ Acerbi, Carlo and. Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk", 2007, p. 4

¹⁰ See for example http://public.wsu.edu/~wldciv/world_civ_reader/ world_civ_reader_1/plato. html

 $^{^{11}}$ Acerbi, Carlo and Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk", 2007, p. 5

funded by liquidating positions in assets held in it or a stock of euros (as per Definition 2, Box 1) held in the portfolio. To determine our liquidity policy, we should find an acceptable balance between the cost of not investing our euros and having to sell off securities at their fundamental or fair price. Also, the proceeds generated from a sale are the liquidation value of a position that we have available for funding our liquidity gap. Hence on one hand, our desire to invest the most euros possible in our portfolio and in the more illiquid assets in order to gain higher return and premium; and on the other hand, our natural limit for tolerance to liquidity risk - the effective bid-offer spread that we are willing to pay if unanticipated cash outflows arise. If the portfolio has high turnover or is used to meet cash outflows or for parking cash inflows, then it is highly exposed to liquidity risk and should be managed under lower risk tolerance levels, using the illiquidity measures elaborated below. On the other hand, held-to-maturity portfolios can bear higher liquidity risk tolerance, but still, the measures expounded below can be used to adjust the liquidity policy as they may herald elevated risk of default before the fundamental value of the holdings has deteriorated.

Overview of the LARC Model

The LARC model stipulates two measures of illiquidity in the markets – Systemic Illiquidity and Idiosyncratic Illiquidity, for the purposes of defining liquidity risk tolerance based on the concepts above. Both are aggregated on issuer and not bond level, using market weights.

The measures are defined as follows:

We posit an OLS regression of the common form:

$$y = \beta X + \varepsilon$$

where y and ϵ are n × 1 vectors, and X is an n × m matrix of regressors, whereby high-frequency data on bid-ask spreads is regressed on lot size, and on lot size and maturity of realized trades, thus providing an adequate proxy of an asset's MSDC.

Although we recognize that the data set may be liable to violating certain OLS assumptions, the high-frequency nature of the data sample allows for shorter periods of input data with ample observations, thus minimizing the impact of the violations discussed afore.

This paper asserts that such an approximation of the MSDC possesses both:

1. Sufficient and reliable predictive power to provide early warning mechanism for market disruptions for given assets and their issuers; and

2. Flexibility to allow for an intuitive way of explicit setting of liquidity risk tolerance.

Definition I. Measure of Systemic Illiquidity in the LARC model is defined as the slope of the regression line in the algebraic space with basis {lot size; bid-ask spread} or the slope of the intersection line of the two regression planes in the algebraic space with basis {lot size; maturity; bid-ask spread}. These lines are generally upward sloping with respect both to lot size and maturity as the liquidity suppliers demand extra premium for increases in inventory costs, depletion of credit line to hold the issuer, cost of sourcing the security, and as adequate yield differences for larger maturities translate into magnified price differences due to longer duration.

Systemic illiquidity shows how distressed the bonds are given the market conditions and reflects both endogenous and exogenous cost of liquidity. One of its merits is that it captures adequately the endogenous component, which ultimately implies the issuer's standing in the capital markets. The greater the slope, the more distressed the security. This measure changes considerably for an issuer or an asset class over time depending on the market perception of the volatility and elevated credit Ffconcerns. An increase in systemic illiquidity implies pending market dysfunction for that asset class or issuer. This measure can be used as a leading indicator as market participants' unwillingness to trade said securities first filters, to a palpable extent, into the dynamic of the effective bid-offer spread.

Definition II. Measure of Idiosyncratic Illiquidity in the LARC model is defined as the bid-offer spread intercept of the regression line in the {lot size; bid-ask spread} space or the intercept of the intersection line of the two regression planes in the {lot size; maturity; bid-ask spread} space.

Idiosyncratic illiquidity shows how illiquid an issuer's securities within an asset class are perceived in general. This measure defines the minimum liquidity premium demanded to trade an issuer's securities and reflects exogenous costs of liquidity such as issue size, number of issues outstanding, frequency of issuance, and presence of an issuer's yield curve. This measure, or an ordering by it, is more stable over time compared to systemic illiquidity as the issuance pattern and market presence of an issuer follow primarily from its business model. Thus sharp increases in idiosyncratic illiquidity and a simultaneous increase in systemic illiquidity portend liquidity freeze-up and pending market collapse for the securities of that issuer.

In sum, in the LARC application, the effective bid-offer spread is used to estimate the MSDC through OLS regression and the slope and intercept of the regression lines are used as measures of systemic and idiosyncratic illiquidity, respectively, of an issuer. An increase in either measure signals

worsening liquidity conditions.

Technically, there are three basic assumptions underlying LARC itself to ensure that the estimate is conservative enough, includes order flow considerations, does not overstate the lot size and provides that liquidity assessment correctly captures the liquidity effect:

Assumption 1. The transactions are signed and matched on a FIFO principle.

Assumption 2. The minimum of the transacted amount is applied as a regressor.

Assumption 3. The directional derivative of the intersection line of the planes in the *{lot size; maturity; bid-ask spread}* space is used for determining the optimal mix of the lot size and maturity components' effect on the effective bid-offer spread; moreover, the derivative also allows for ranking. In the next part of this Section we will briefly describe the software implementation used within LARC.

Based on the definitions stated above, the following problems can be formulated:

Problem 1. Using the above constructs and formalisms devise a practical application for extracting information embedded in transaction costs to give liquidity assessment of assets in the investment universe of an institution, and identify in advance the assets' likelihood to be adversely affected.

Problem 2. Determine a method for assessment and implementation of an instituion's optimal liquidity policy based on its investment universe and prevailing market conditions.

In this paper the first problem is addressed in detail, whereas the second one is an area of future research and development.

The software solution used to solve Problem 1 consists of a database, core, and interface.

The database includes the externally fed input – in the case studies below: MTS Data, and all the tables and queries that apply *Assumption 1* and *Assumption 2* as previously defined, so as to derive the bid-ask spread values, relevant lot sizes and maturity buckets. Also, the design of the database specifically addresses asset class ambiguities and allows for aggregations of the trade data on the issuer level within an asset class.

The values of the systemic and idiosyncratic illiquidity for each issuer are calculated depending on the period and asset classes selected in the input phase. Then, depending on the constraints set in the input phase, a list of accepted and a list of rejected issuers is generated within a so-called Preliminary Report and visualizations of the effective bid-ask spread dynamic are rendered in both the two-dimensional *{lot size; bid-ask spread}}* and three-

dimensional *{lot size; maturity; bid-ask spread}* case. These are visualized in the interface of LARC application.

The model does not adjust for violations in OLS assumptions and thus excessively long periods of data are not advised. Yet, since the tool under consideration assesses liquidity conditions at a given point in time, it is natural to prefer using conditional input, i.e. the most recent data available, and to only go back far enough to assure sufficiency of data. Hence the aspiration to use as high-frequency data as available in the case studies below.

Section 2: Case Studies

In this section we look into a case study application of LARC on MTS data. In the words of its provider, MTS Group¹², the data are "[r]eal-time tradable prices for European government, quasi-government and covered bonds. MTS is the leading electronic system delivering real-time tradable prices across the entire European government, quasi-government and covered bond market."¹³ As this case study focuses on Euro-denominated fixed-income instruments, we consider this to be one of the most pertinent data sources. Moreover, the frequency and nature of the data offered fits the purpose of the case study. The period used – Jan. 2007 – Nov. 2009, provides testing ground s spanning both normal and stressed market conditions, with various asset classes and issuers being affected. The data set consists of over 980 000 executed trades of various lot sizes covering virtually the whole spectrum of euro-denominated bond issuance.

First, we look into liquidity conditions in the Pre-Crisis Period where market friction was practically non-existent and the endogenous costs of illiquidity were minuscule. Then we consider the liquidity conditions in the Run-up to Lehman Brothers' Failure, which most severely affected structured, assetbacked and covered securities, as well as bonds of financial institutions. Yet, the model manages to identify the imminent housing market collapse following the summer of 2007. For both periods we look into Nationwide Building Society issuer as a case in point.

Next, we assess the liquidity conditions in the Aftermath of Lehman Brothers' Failure, where the market starts establishing a "new normal" in terms of liquidity premia. In this period we turn to Government and Quasi-government issuers that begin to be affected. The two particular cases considered are Ireland and Kredit fuer Wiederaufbau (KfW).

¹² http://www.mtsmarkets.com

¹³ http://www.mtsmarkets.com/Products/MTS-Data

Finally, we look into the Run-up to the Eurozone Sovereign Debt Crisis, as worsening liquidity conditions beyond housing market distress creep further into the Government and Quasi-Government asset classes, foreshadowing the exacerbation of market and credit risk that ensues.

We present graphs of the liquidity dynamics of each member (issuer) and a preliminary report, applying liquidity constraints for the period, and then we compare the results.

It should be noted that the algorithm is generic and can be back-tested against any data set. Therefore, its applicability in other time frames and for different securities in other markets is valid. The most interesting assertion that we try to prove is that *liquidity conditions deteriorate prior to increased volatility and worsening credit profile and thus, worst-hit victims of each crisis stand out in terms of illiquidity as measured by LARC (especially systemic illiquidity) even when the input period precedes the formal events.* Hence, LARC proves to possess sufficient predictive power and potential to be used preemptively.

First, we specify the minimum number of transaction on which we would base our statistical analysis. Obviously, the fewer the transactions, the less reliant we can be on the estimate. Also, fewer transactions by themselves imply lack of liquidity. Secondly, we specify our maximum values for systemic and idiosyncratic illiquidity. This can be done in absolute terms – by stating the maximum amount of cents (or units of a bonds price quoted per 100), or in relative terms - by stating the percentage of least liquid issuers (either by the systemic or idiosynctratic criterion) that we wish to exclude. This approach allows for both concrete illiquidity ceilings and for retention of the adequately liquid portion of the market depending, on our liquidity risk aversion.

For all periods, the following filtering criteria are used as constraints for issuer eligibility:

- Minimum Number of Transactions 10;
- Relative Idiosyncratic Illiquidity 25%;
- Relative Systemic Illiquidity 25%;
- Absolute Idiosyncratic Illiquidity 2 cents;
- Absolute Systemic Illiquidity 2 cents.

These criteria have been chosen heuristically: the minimum number of transactions of 10 to show that even with fewer observations than necessary for the application of the Central Limit Theorem the results are plausible, and the model does not fail even with less-than-perfect inputs; the relative systemic and idiosyncratic illiquidity limit of 25% for exclusion of the upper quartile of the most illiquid assets; and the absolute idiosyncratic and systemic illiquidity of 2 cents - as a relatively relaxed constraint, to show that in

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distressed conditions some issuers breach even this constraint. As a rule of thumb, we assume that the predictive power of the model holds for a third of the length of the period used as input data, i.e. if three months of data is used for the estimation, the eligibility or ineligibility designation by LARC for a given issuer will be expected to hold for the next one month. In the analysis that follows, certain periods are identified which point out that increased illiquidity as measured by LARC precedes increased adverse movements in the spread of an issuer's bonds against the German benchmark, and also precedes elevated market volatility. Moreover, LARC would accept issuers as eligible even with turbulent input in cases where spread tightening and reduced volatility ensue, showing that the predictive power of LARC is valid both ways.

The Preliminary Report following a graphic representation of the bid-offer dynamic lists all the issuers, grouping them in the following categories:

- · Accepted issuers, in the two-dimensional and three-dimensional case;
- · Rejected issuers, grouped by the limit breached;
- · Not Traded issuers.

The preliminary report gives the list of issuers eligible according to the liquidity risk tolerance set. It also allows for analysis of the amount and type of illiquidity characterizing each issuer and for certain inferences on asset level. In the cases below, for brevity of exposition, excerpts with the issuer under consideration are presented rather then the full preliminary reports.

It is important to note that LARC can be used to establish buy as well as sell signals and the following case studies will also identify periods when it is prudent to invest again in the issuer. The rule of thumb used is that the horizon for the prediction is roughly 1/3 of the length of the period on which the prediction is based. For simplicity, the cases presented below are based on data spanning three months and thus the prediction is valid within the next one month.

Pre-Crisis and Run-up to Lehman Brothers' Failure: Nationwide Building Society (NBS)

In Figure 1, we can see that NBS trades at tight spreads during the first half of 2007 but spreads widen significantly after June.

NBS: YIELD SPREAD DYNAMICS VS. GERMANY JAN 2007 - DEC. 2007



For input period Feb 1st – May 1st, 2007, LARC accepts the issuer with the aforementioned constraints as evidenced by the preliminary report shown in Figure 2 and the 3D bid-offer dynamic shown in Figure 3. The result is expected as market conditions remain favorable for the Structured Bonds market.

Figure 2

PRELIMINARY REPORT, STRUCTURED BONDS,

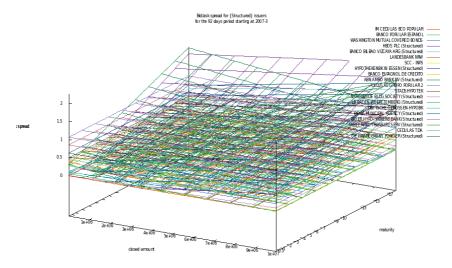
1ST FEB – 1ST MAY, 2007

Structured	NATIONWIDE BLDG SOCIETY
Structured	WASHINGTON MUTUAL COVERED BONDS
Structured	CAJA AHORROS BARCELLONA
Structured	CAJA AHORROS BARCELLONA
Structured	CASSA DEPOSITI E PRESTITI
Structured	CASSA DEPOSITI E PRESTITI
Total	20

	Rejected Issuers		
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)		
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity		
Filtered Component	Bid-offer Spread Intercept		
Required Value	Required Value -0.012001		

riequired value	-0.012001		
Asset Class	Issuer	Actual Value	Deviation
Structured	SCC - INPS	-0.012001	100 %
Structured	DEPFA PFANDBRIEFBANK	0.127476	-1,062.211 %
Structured	DEUTSCHE GENOSSEN-HYPOBK	-0.011129	92.734 %
Structured	EUROHYPO AG	0.017481	-145.663 %
Structured	HBOS PLC	0.008354	-69.611 %
Structured	BANCO BILBAO VIZCAYA ARG	0.081010	-675.027 %
Structured	WASHINGTON MUTUAL COVERED BONDS	0.539721	-4,497.3 %
Structured	DEXIA MUNICIPAL AGENCY	0.185508	-1,545.771 %
Total	8		

3D RENDERING OF EFFECTIVE BID-OFFER SPREAD, STRUCTURED BONDS, $\mathbf{1}^{ST}$ FEB – $\mathbf{1}^{ST}$ MAY, 2007



For input period Mar 1st – June 1st, 2007, LARC rejects NBS with the aforementioned constraints. NBS still trades at tights vs. Germany but liquidity is deteriorating and LARC identifies a sell opportunity prior to the housing market's collapse (see Figure 4 and Figure 5).

Figure 4

PRELIMINARY REPORT, STRUCTURED BONDS, 1ST MAR – 1ST JUNE, 2007

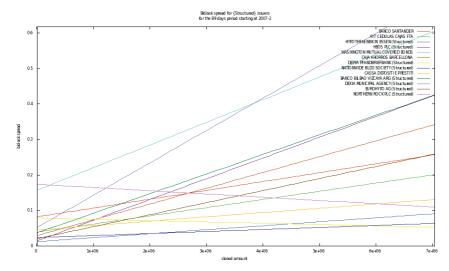
Structured	CAJA AHORROS BARCELLONA	
Structured	CASSA DEPOSITI E PRESTITI	
Structured	CASSA DEPOSITI E PRESTITI	
Structured	CAJA AHORRO MONTE MADRID	
Structured	CAJA AHORRO MONTE MADRID	
Total	18	

нe	eci	led	ISS	ue	rs

Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity
Filtered Component	Bid-offer Spread Intercept
Required Value	-0.021001

Asset Class	Issuer	Actual Value	Deviation
Structured	NATIONWIDE BLDG SOCIETY	-0.015588	74.225 %
Structured	IM CEDULAS BCO POPULAR	-0.005262	25.056 %
Structured	ABBEY NATL TREASURY SERV	0.105000	-499.976 %
Structured	SCC - INPS	-0.021001	100 %
Structured	BANCO BILBAO VIZCAYA ARG	0.233544	-1,112.061 %
Structured	HBOS PLC	0.251673	-1,198.386 %
Structured	DEXIA MUNICIPAL AGENCY	0.176121	-838.631 %
Structured	HYPOTHEKENBK IN ESSEN	0.166759	-794.053 %
Total	8		

2D RENDERING OF EFFECTIVE BID-OFFER SPREAD, STRUCTURED BONDS, 1ST MAR – 1ST JUNE, 2007 – NBS CLEARLY STANDS OUT BOTH IN TERMS OF SYSTEMIC (SLOPE) AND IDIOSYNCRATIC (INTERCEPT) ILLIQUIDITY.



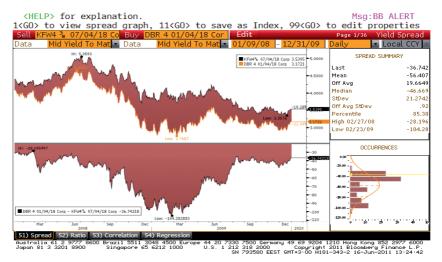
Source: Bloomberg LP

It is important to note that in this as in all the following cases, the outcome is dependent on the criteria chosen in the input phase. In the event of constraints more relaxed than the ones set above, all traded issuers may be accepted. In the other extreme, with very conservative constraints – say very large number of transactions and tight bid-ask spread, all issuers may end up being rejected as not liquid enough.

Aftermath of Lehman Brothers' Failure: KfW and Ireland

In Figure 6 the spread of KfW, an issuer with explicit guarantee from the German government, vs. the German Bund is presented.

KfW: YIELD SPREAD DYNAMICS VS. GERMANY MAR. 2008 - DEC. 2009



Source: Bloomberg LP

Asset Class

Issuer

A widening of the spread, albeit volatile, can clearly be identified for the period Q4, 2008 – Q1, 2009. This is due to the talks that the Hypo Real Estate (HRE) bailout would go through KfW.

For the input period Nov. 1st, 2008 – Feb. 1st, 2009 the issuer is rejected as not covering the liquidity requirement (see Figure 7 and Figure 8).

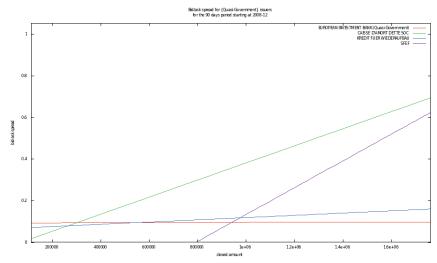
Figure 7

PRELIMINARY REPORT, QUASI-GOVERNMENT BONDS, 1ST NOV., 2008 – 1ST FEB, 2009

LARC Preliminary Report

Quasi-Government	EUROPEAN INVESTMENT BANK		
Total	1		
	Rejected Issuers		
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturit	ty)	
Description	Lists all issuers with greater than the specified Idiosyncr wrt Lot Size and Maturity	ratic Illiquidity measure in r	elative terms (%)
Filtered Component	Bid-offer Spread Intercept		
Required Value	0.123513		
Asset Class	Issuer	Actual Value	Deviation
Quasi-Government	KREDIT FUER WIEDERAUFBAU	0.123513	100 %
Quasi-Government	SFEF	0.150021	121.462 %
Total	2		
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size)		
Description	Lists all issuers with greater than the specified Idiosyncr wrt Lot Size	ratic Illiquidity measure in r	elative terms (%)
Filtered Component	Bid-offer Spread Intercept		
Required Value	0.190469		
Asset Class	Issuer	Actual Value	Deviation
Quasi-Government	BARCLAYS BANK PLC	1.610000	845.282 %
Quasi-Government	KREDIT FUER WIEDERAUFBAU	0.190469	100 %
Total	2		

2D RENDERING OF EFFECTIVE BID-OFFER SPREAD, QUASI-GOVERNMENT BONDS, 1ST NOV., 2008 – 1ST FEB, 2009



On the other hand, for the input period Dec. 1st, 2008 – March 1st, 2009, while KFW still trades at wides vs. Germany, liquidity improved and LARC identifies a buy opportunity (see Figure 9 and Figure 10). Indeed within the next month the spread embarks on a tightening trend (see Figure 6).

Figure 9

PRELIMINARY REPORT, QUASI-GOVERNMENT BONDS, 1ST DEC., 2008 – 1ST MAR., 2009 LARC Preliminary Report

Accepted Issuers

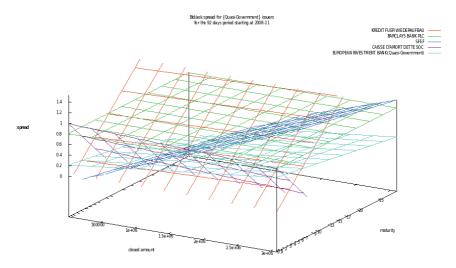
Asset Class

Issuer

Addet Clado	100461		
Quasi-Government	KREDIT FUER WIEDERAUFBAU		
Total	1		
	Rejected Issuers		
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)		
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity		
Filtered Component	Bid-offer Spread Intercept		
Required Value	0.026287		
Asset Class	Issuer	Actual Value	Deviation
Quasi-Government	SFEF	0.153708	584.73 %
Quasi-Government	EUROPEAN INVESTMENT BANK	0.026287	100 %
Total	2		
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size)		
Description	Lists all issuers with greater than the specified Idiosyncratic II wrt Lot Size	lliquidity measure in r	elative terms (%)
Filtered Component	Bid-offer Spread Intercept		

riequired value	0.003300		
Asset Class	Issuer	Actual Value	Deviation
Quasi-Government	EUROPEAN INVESTMENT BANK	0.093104	142.431 %
Quasi-Government	KREDIT FUER WIEDERAUFBAU	0.065368	100 %
Total	2		

3D RENDERING OF EFFECTIVE BID-OFFER SPREAD, QUASI-GOVERNMENT BONDS, 1ST DEC., 2008 – 1ST MAR., 2009



In Figure 11, the spread of Ireland vs. the German Bund for the period Jan. 1st, 2008 – July 1st, 2009 is presented. Clearly the spread widens after Lehman Brothers' failure.

Figure 11 **DYNAMICS OF IRELAND VS. GERMANY JAN 2008 – JULY 2009**



Source: Bloomberg LP

DISCUSSION PAPERS

Figure 12

-675.027 %

4,497.3 %

-1,545.771 %

With input Jan. 15th – April 15th, 2009, although Ireland is still at wides vs. Germany, LARC identifies a calming period following the initial onset of the banking crisis (see Figure 12 and Figure 13). This is a buy opportunity with for our 1 month horizon with good carry and tightening potential.

PRELIMINARY REPORT, GOVERNMENT BONDS, 15TH JAN. – 15TH APR., 2009

Structured	NATIONWIDE BLDG SOCIETY
Structured	WASHINGTON MUTUAL COVERED BONDS
Structured	CAJA AHORROS BARCELLONA
Structured	CAJA AHORROS BARCELLONA
Structured	CASSA DEPOSITI E PRESTITI
Structured	CASSA DEPOSITI E PRESTITI
Total	20

Rejected Issuers

Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)		
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity		
Filtered Component	Bid-offer Spread Intercept		
Required Value	-0.012001		
Asset Class	Issuer	Actual Value	Deviation
Structured	SCC - INPS	-0.012001	100 %
Structured	DEPFA PFANDBRIEFBANK	0.127476	-1,062.211 %
Structured	DEUTSCHE GENOSSEN-HYPOBK	-0.011129	92.734 %
Structured	EUROHYPO AG	0.017481	-145.663 %
Structured	HBOS PLC	0.008354	-69.611 %

BANCO BILBAO VIZCAYA ARG

DEXIA MUNICIPAL AGENCY

WASHINGTON MUTUAL COVERED BONDS

Structured

Structured

Structured

Total

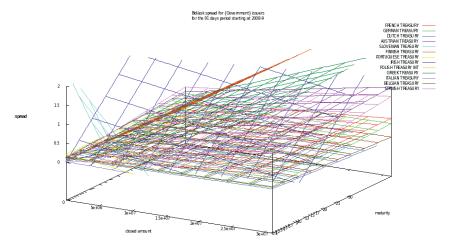
Figure 13
3D RENDERING OF EFFECTIVE BID-OFFER SPREAD, GOVERNMENT

BONDS, 15TH JAN. - 15TH APR., 2009

0.081010

0.539721

0.185508



With input Sept. 15th – Dec. 15th, 2009, Ireland is excluded as eligible issuer according to our liquidity risk criteria, just before its spread vs. Germany spikes at the end of December following Fine Gael's resistance to an Anglo-Irish bailout and the onset of recession in Q1 2009. Thus LARC provides an early warning signal while the issuer can still be liquidated at better levels (see Figure 14 and Figure 15).

PRELIMINARY REPORT, GOVERNMENT BONDS, 15^{TH} SEPT. – 15^{TH} DEC., 2009

		•
Government	FRENCH TREASURY	
Government	BELGIAN TREASURY	
Government	BELGIAN TREASURY	
Government	GERMAN TREASURY	
Government	GERMAN TREASURY	
Total	16	

Rejected Issuers

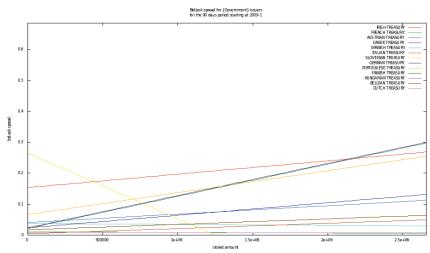
Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)	
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity	
Filtered Component	Bid-offer Spread Intercept	
Required Value	-0.077997	

Asset Class	Issuer	Actual Value	Deviation
Government	GREEK TREASURY	-0.044186	56.651 %
Government	ITALIAN TREASURY	-0.021037	26.972 %
Government	POLISH TREASURY INT	-0.077997	100 %
Government	IRISH TREASURY	-0.035073	44.967 %
Total	4		

Figure 15

Figure 14

2D RENDERING OF EFFECTIVE BID-OFFER SPREAD, GOVERNMENT BONDS, 15TH SEPT. – 15TH DEC., 2009



Run-up to the Eurozone Sovereign Debt Crisis: Greece

For our discussion on the Run-up to the Eurozone Sovereign Debt Crisis we will look into the case of Greek government bonds. We will show that although Euroarea governments enjoyed convergence in bond yields, actually the overlooked differences in liquidity were present even before the bonds traded as distressed.

In Figure 16, the spread of Greek bonds vs. the German Bund for the period Jan. 1st, 2009 – Dec 31st, 2009 is presented. The spread widens post Lehman Brothers' failure, but then tightens again until problems intrinsic to bonds of the Eurozone Periphery begin emerging towards the end of the period under consideration.

Figure 16 **DYNAMICS GREECE VS. GERMANY JAN 2009 – DEC. 2009**



Source: Bloomberg LP

Similarly to the case of Ireland, with input April 1st – July. 1st, 2009, Greece still trades at wides vs. Germany. Yet, LARC signals improving liquidity following the initial onset of the banking crisis (see Figure 17 and Figure 18). This clearly is a buy opportunity with for our 1 month horizon with good carry and tightening potential, which is evidenced in the yield spread graph (Figure 16).

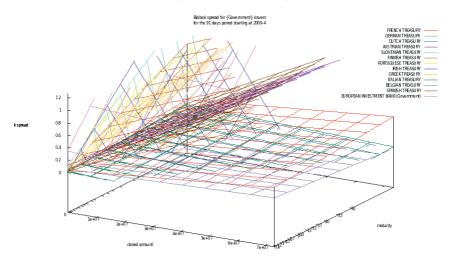
DP/88/2012

PRELIMINARY REPORT, GOVERNMENT BONDS, 1ST APRIL 1ST JULY., 2009

LARC Preliminary Report Accepted Issuers

Asset Class	Issuer	
Government	ITALIAN TREASURY	
Government	GREEK TREASURY	
Government	PORTUGUESE TREASURY	П
Government	DUTCH TREASURY	

Figure 18 3D RENDERING OF EFFECTIVE BID-OFFER SPREAD, GOVERNMENT BONDS, 1ST APRIL 1ST JULY., 2009



With input July 1st – September 1st, 2009, Greece is excluded as eligible issuer according to our liquidity risk criteria. Despite Greece still trading relatively tight vs. Bunds, LARC identifies the draining liquidity as fewer and fewer traders are willing to keep Greek bonds on their books, and advises a sale (see Figure 19 and Figure 20). This sell signal comes some one month and a half prior to the market meltdown following the unveiled faulty Greek stats by Eurostat, leaving plenty of time for action.

PRELIMINARY REPORT, GOVERNMENT BONDS, 1ST JULY – 1ST SEPT., 2009

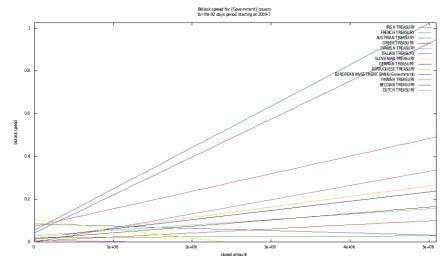
Rejected Issuers

Filter	Idiosyncratic Illiquidity, Relative (wrt Lot Size and Maturity)	
Description	Lists all issuers with greater than the specified Idiosyncratic Illiquidity measure in relative terms (%) wrt Lot Size and Maturity	
Filtered Component	Bid-offer Spread Intercept	
Required Value	0.012171	

Asset Class	Issuer	Actual Value	Deviation
Government	IRISH TREASURY	0.078118	641.837 %
Government	GREEK TREASURY	0.029793	244.787 %
Government	AUSTRIAN TREASURY	0.037823	310.763 %
Government	SLOVENIAN TREASURY	0.012171	100 %
Total	4		

Figure 20

2D RENDERING OF EFFECTIVE BID-OFFER SPREAD, GOVERNMENT BONDS, 1ST JULY – 1ST SEPT., 2009



Note: The steep slope for EIB (green) comes with the rumors that a Greek rescue is to be mounted through the bank; second steepest is Greece and the top three issuers with the highest Systemic Illiquidity for the period are completed with Ireland.

Last but definitely not least, we will use the above chart to illustrate and clarify some characteristics of the regression lines. We begin with what may seem like an obvious 'anomaly', namely, the negative slope for Slovenian

Treasury. Other things equal, we would expect to see positive slopes across the board – however, in this case, the negative slope is an artifact of the data. Since there were not enough data points for Slovenia (and those few we had are clustered mostly in the lower-left section of the chart), the regression result is not reliable. There is a potential workaround for this, to be discussed below.

Next, we tackle something more important – the relative magnitude of the slopes and the intercepts. The Greek Treasury stands out as one of the steepest slopes, as expected – but it has an intercept lower than, say, the Irish Treasury. A steeper slope indicates a higher level of distress for an issuer, whereas a higher intercept indicates shallower trading in that issuer. Thus, the correct reading here is not that Greece is a better credit than Ireland, but rather that Greece is traded more heavily than Ireland. Since our algorithm assigns more weight to the slope than the intercept, this interpretation is consistent with Ireland being the better credit, overall.

Finally, we'd like to draw attention to a future modeling improvement. Namely, a step-wise linear regression algorithm can be designed that would have superior performance (based on already observed work-in-progress) compared to the OLS algorithm. For example, this would produce a more reliable result for Slovenia – among others – and would, in general, more accurately capture the dynamics of sub-periods within the sample period. This could be very important since, even within a relatively short time-span of 3 months, an event can occur that can portend a 'regime switch' for certain issuers – and therefore we would need to model the data *ante* and *post* said event.

Concluding remarks

We have proposed an inovative way of applying the formalism proposed by Acerbi and Scandolo. The application, LARC software, draws heavily on market microstructure assumptions and makes heavy use of the dynamics of effective bid-offer spreads. This approach exibits reliability and sufficient predictive power when tested with real euro-bond market data from MTS. Specifically, the way the effective bid-ask spreads are constructed, is novel and captures very efficiently the dynamic of real-world trading – as well as reflects and incorporates 'soft' market perceptions and news in-between trades.

The proposed liquidity measures – namely systemic and idiosyncratic illiquidity – do indicate ensuing elevated levels of market volatility and possible market failure. The concrete case studies considered in the paper show that the algorithm can be adequately applied for various fixed income instruments under different market conditions – from the Pre-Crisis period through the Lehman Brothers' Failure to to the European Sovereign Debt Crisis.

As the algorithm is generic, possible areas for future development include:

- 1) Implementation of an institution's optimal liquidity policy. This can be used as an overlay to a traditional Markowitz optimization solution by maturity sectors for optimal issuer selection to maximize spread premium, without jeopardizing liquidity beyond the investor's tolerance level. Alternatively, possibilities include an optimization which could be a self-contained solution for maximization of return in liquidity portfolios, where, for lack of adequate assessment of their market liquidity, many issuers and instruments are excluded from the investment universe.
- 2) Extension into other asset classes, such as equities in the 2-dimentional case {lot size; bid-ask spread} or residual value in leasing and other contracts, with {number; age; residual value} as factors determining liquidation value in the 3-dimensional case.
- 3) Liquidity-at-Risk as a full-blown model, effectively mirroring and complementing the Value-At-Risk framework widely implemented in the EU and North America. This has additional relevance in the context of Basel III, and would be directly applicable not only to buy-side investors, but to any financial institution that needs proper and timely management of its cash inflows vs. outflows.

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BOX 1: Definitions of concepts underlying the model

Below we list the definitions upon which we build the mathematical model underlying LARC, several of them stated verbatim from Acerbi and Scandolo:

Definition 1. Let A be an Asset, x_i (where $i \in \{1, 2\}$) be the quantity of asset A traded.

An Asset A is a good traded in the market with prices given by a map

$$m: R_* \rightarrow R^{14}$$

called its 'Marginal Supply-Demand Curve' (MSDC) satisfying

- 1. $m(x_1) \ge m(x_2)$ if $x_1 < x_2$
- 2. m is cadlag for x < 0
- 3. m is ladcag for $x > 0^{15}$

Note that the definition of an asset above does not assume the ideal situation that markets are frictionless. There is a bid-ask spread even for the smallest traded quantities, which reflects the exogenous liquidity costs (Definition 18).

The continuity conditions for the process are imposed to ensure that the limits at 0^- and 0^+ exist and that the bid and ask prices deviate from the fundamental fair value price of the asset in a way consistent with the effect of order flow (see BOX 2: Market Microstructure concepts applied) - a sell order prompts the liquidity supplier to lower his/her fundamental price expectation for the next period of trading, while a buy order prompts him to raise this expectation. Moreover, these continuity conditions allow for our formal definition of proceeds from the sale of an asset below, as the price function in Definition 5 is integrable 16 . To measure the impact on the value, we need to define an asset which will serve as the unit in which we convert our holdings. Thus we define:

Denition 2. Let A be an Asset, x be the quantity of asset A traded, m be the map defined in Definition 1, giving the MSDC.

The "euro" is a special Asset A_0 whose MSDC $m_0(x)$ is identically equal to 1 for all $x \in R_*$. It is the currency which is delivered cash in any trade. The "market" is a finite collection of assets $\mathfrak{M} = \{A_i \mid i=0,...,N\}$, where $N \geq 1$ containing the euro.¹⁷

¹⁴ Notation: $R_* := R \setminus \{0\}$

¹⁵ Acerbi, Carlo and Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk", 2007. p.5

It is piecewise continuous in the open interval $(-\infty; +\infty)$

 $^{^{17}}$ Acerbi, Carlo and Giacomo Scandolo, "Liquidity Risk Theory and Coherent Measures of Risk", 2007, p. 6 $\,$

The "euro" is the ultimate means that we use to meet our liquidity needs and thus it is unaffected by liquidity risk and has an MSDC identically equal to 1. It is the asset in the form of which we receive our proceeds from closing a position.

Except for the euro, other assets trade at a bid-ask spread and below follow formal definitions of this spread. The definition of the best bid-ask spread follows heuristically from the definition of an asset.

Definition 3. Let m be the map defined in Definition 1, giving the MSDC. We call $m^+ := m(0^+)$ the 'best bid' and $m^- := m(0^-)$ the 'best ask'. The positive quantity $\delta m := m^- - m^+ \geq 0$ is called the 'bid-ask spread'. We denote $\mathcal M$ the set (actually, a convex cone) of all possible MSDC. ¹⁸

This implies the market is presumed to be better able to absorb smaller quantities. Certainly the above example is an oversimplification as there is some inescapable fixed transaction cost (even if only measured by the time employees need to handle the transaction), no matter the quantity. Yet, it is a good starting point.

Furthermore, such a construct does not necessarily suggest that trading two lots of half the holding each would result in lower cost, as, when order flow effects are taken into account, the fundamental value of the leftover units may change, and the bid quote would be lower, as the liquidity supplier has adjusted his/her expectations. This does not imply a lowering of the ask quote, though, and the bid-ask spread is likely to have widened.

Definition 4. Let A be an Asset, x be the quantity of asset A traded, m be the map defined in Definition 1, giving the MSDC.

The price m(x) for x > 0 (x < 0) is the bid (ask) price associated to a quote of size dx, expressed in units of the asset.¹⁹ In this paper, we shall call the absolute difference between two contemporaneous or consecutive bid, $m(x_1)$, and ask, $m(x_2)$, prices, the effective bid-ask spread for the quantity $min\{x_1, x_2\}$.

As discussed, the amount of proceeds in euro depends on the realization of the price we receive. More formally we define proceeds below.

Definition 5. Let A be an Asset, x and s be the quantities of asset A traded, m be the map defined in Definition 1, giving the MSDC.

The "Proceeds" P(s) for transaction of $s \in R$, assets are

$$P(s) := \int_{0}^{s} m(x)dx \tag{1.1}$$

¹⁸ Ibid., p. 6

¹⁹ Ibid., p. 6

Note that in this definition P(s) is the total amount we receive when we sell s > 0 assets and minus the total amount we pay when we buy |s| assets when s < 0.²⁰

Hence proceeds are the value resulting from the sum of the values of units of the asset, each eliciting a specific market realization of the price function. These prices, on their part, are given by the Supply-Demand Curve of the asset.

Definition 6. Let A be an Asset, s be the quantity of asset A traded, and P (s) be the Proceeds for the transaction, as per Definition 5.

The "Supply-Demand Curve" S(s) (SDC) for transaction of $s \in R_*$ assets is:

$$S(s) := \frac{P(s)}{s} \tag{1.2}$$

For s > 0, S(s) is the average unit price of the sale of s assets. For s < 0, S(s) is minus the average unit price of the purchase of |s| assets.²¹

The assets traded are held as positions in portfolios of assets and below we give a formal definition of a portfolio, which we will use further in this work.

Definition 7. A "Portfolio" is a vector $\mathbf{p} = (\mathbf{p}_0, ..., \mathbf{p}_N) \in \mathbb{R}^{N+1}$, where \mathbf{p}_0 is the "portfolio liquidity" and $\vec{p} := (\mathbf{p}_1, ..., \mathbf{p}_n) \in \mathbb{R}^N$ is the "assets' position". We will say that \mathbf{p} is "long-", "flat-" or "short- asset A_k " if $\mathbf{p}_k > 0$, $\mathbf{p}_k = 0$ or $\mathbf{p}_k < 0$ respectively.²²

Let $\mathcal{P} = \mathbb{R}^{N+1}$ be the space of all portfolios.

Definition 8. Let \mathcal{P} be the space of all portfolios and $\mathbf{p} \in \mathcal{P}$ be a portfolio vector as per Definition 7; and A be an Asset, x be the quantity of asset A traded, and m be the map defined in Definition 1, giving the MSDC.

The "Liquidation" $L(\mathbf{p})$ of a portfolio $\vec{p} \in \mathcal{P}$ is defined by

$$L(\mathbf{p}) := \sum_{i=0}^{N} P_i(p_i) = p_0 + \sum_{i=1}^{N} \int_{0}^{p_i} m_i(x) dx$$
 (1.3)

The quantity $L(\mathbf{p})$ represents the total proceeds coming from a liquidation of the portfolio.²³

²⁰ Ibid., p. 7

²¹ Ibid., p. 7

²² Ibid., p. 7

²³ Ibid., p. 7

This formula stipulates that the proceeds we generate from shifting the assets to cash equal the amount of euro we hold (as there is no transaction cost incurred for it), plus the proceeds we generate from closing the positions at the prevailing market conditions taking a price realization determined by the size and other attributes of our assets, and paying the liquidity premium in the form of effective bid-offer spread.

Definition 9. Let \mathcal{P} be the space of all portfolios and $\mathbf{p} \in \mathcal{P}$ be a portfolio vector as per Definition 7, and $\mathbf{a} \in \mathbb{R}^+$.

A "liquidity policy" $\mathcal L$ is any closed convex subset $\mathcal L \subseteq \mathcal P$ of the space of portfolios satisfying

- 1. $\mathbf{p} \in \mathcal{L} \Rightarrow \mathbf{p} + a \in \mathcal{L}, \forall a > 0$
- 2. $\mathbf{p} = (p^0, \vec{p}) \in \mathcal{L} \Rightarrow (p^0, \vec{0}) \in \mathcal{L}$

A liquidity policy is therefore a type of constraint on a portfolio \mathbf{p} for which the portfolio liquidity p_0 is never too large and the absolute value $|p_k|$ of any illiquid asset (k > 0) is never too small.²⁴

Liquidity policy and explicit stipulation of it is the key endeavor of this paper. It should ensure that prices realized are not far from the mark-to-market (MtM) policies employed by the holder of the assets. Below we define formally some MtM policies.

Definition 10. Let \mathcal{P} be the space of all portfolios as per Definition 7; and \mathcal{L} be a liquidity policy as per Definition 9.

Given two portfolios \mathbf{p} , $\mathbf{q} \in \mathcal{P}$, we say that q is "attainable" from \mathbf{p} , and we write $\mathbf{q} \in Att(\mathbf{p}) \subseteq \mathcal{P}$ if $\mathbf{q} = \mathbf{p} - \mathbf{r} + \mathcal{L}(\mathbf{r})$ for some $\mathbf{r} \in \mathcal{P}$.

Definition 11. Let **p** be a portfolio and \mathcal{P} be the set of all portfolios as per Definition 7; and \mathcal{L} be a liquidity policy as per Definition 9.

The "MtM policy" or simply the "Value" of a portfolio \mathbf{p} under the liquidity policy \mathcal{L} , is a map $V^{\mathcal{L}}: \mathcal{P} \to R$ defined²⁶ by

$$V^{\mathcal{L}}(\mathbf{p}) := \sup\{U(\mathbf{q}) \mid \mathbf{q} \in \mathsf{Att}(\mathbf{p}) \cap \mathcal{L}\}$$
 (1.4)

Definition 12. Let **p** be a portfolio and \mathcal{P} be the set of all portfolios as per Definition 7; and let the U and U' be MtM policies as per Definition 11.

The "Uppermost MtM policy" (see Definition 11), $U(\mathbf{p})$, of a port-folio $\mathbf{p} \in \mathcal{P}$ is defined²⁷ by

$$U(\mathbf{p}) := p_0 + \sum_{p_i > 0} m_i^+ p_i^- + \sum_{p_i < 0} m_i^- p_i^-$$
(1.5)

²⁴ Ibid., p. 9

²⁵ Ibid., p. 8

²⁶ Ibid., p. 10

²⁷ Ibid., p. 8

Using the operator²⁸ U' based on the concept of marking-to-mid market,

$$U'(\mathbf{p}) := \sum_{i=0}^{N} m_i(0) p_i$$
 (1.6)

The Liquidation and the Uppermost MtM Policy are the two extreme cases in portfolio valuation and determine the Liquidation Cost of a portfolio.

Definition 13. Let **p** be a portfolio and \mathcal{P} be the set of all portfolios as per Definition 7.

The liquidation cost,
$$C(\mathbf{p})$$
 of a portfolio²⁹ $\mathbf{p} \in \mathcal{P}$ is
$$C(\mathbf{p}) := U(\mathbf{p}) - L(\mathbf{p}) \in R_{+}$$
(1.7)

Depending on their role in the market that ultimately affects their liquidity policy as well, the market participants are classified in one of the following two groups in the Market Mictrostructure Approach to liquidity risk:

Definition 14. Liquidity Demanders: Traders who want their orders (buy or sell) to be executed immediately at posted prices.³⁰ (e.g. a passive portfolio manager who tries to minimize his tracking error.³¹)

Definition 15. Liquidity Suppliers³²:

- 1. Traders who post quotes and quantities at which liquidity deman ders' orders can be executed.
- 2. These traders do not have specifc reasons to trade (e.g. portfolio rebalancing) or can afford to delay the execution of their order.

Definition 16. Order Flow (or Order Imbalance): Transaction sizes with a sign (+/-) from the point of view of the liquidity demander. It is key because buy and sell orders initiated by liquidity demanders do not execute at the same price; hence transaction prices are related to the direction of order flow. The extent to which order flow moves prices is a measure of market illiquidity.³³

As we shall see in greater detail in *BOX 2: Market Microstructure Concepts Applied*, the sign of the order a liquidity supplier receives (bid or ask) affects the price realization through adjustment of the price expectations of the liquidity supplier in the next round of trading.

²⁸ Ibid., p. 12

²⁹ Ibid., p. 8

³⁰ Foucault, Thierry (HEC, Paris), "Market Microstructure Approach", 2007, p.8

³¹ Ibid., p. 8

³² Ibid., p. 8

³³ Ibid., p. 10

Definition 17. Trading Costs: They constitute a measure of market illiquidity. The higher trading costs are, the lower market liquidity is.³⁴ The Liquidation cost in Definition 13 is the maximum transaction cost for turning the assets in a portfolio into euros as per Definition 2.

The cost of sourcing liquidity can be subdivided into exogenous and endogenous cost of liquidity. Exogenous costs cover the risks a liquidity supplier incurs for the possibility of trading against an informed market participant, as well as fixed costs related to processing the trade.

Definition 18. Exogenous costs of liquidity are the result of varying bidask spreads.³⁵ In the analysis presented and in the LARC application for assessing market liquidity risk, these costs affect both the idiosyncratic illiquidity and systemic illiquidity characteristic of an issuer.

Endogenous costs augment effects that result from the reluctance of market participants to hold an asset due to elevated market and credit risk levels for the asset, or, alternatively, dearth of the asset due to high demand, increased cost of sourcing the asset and the opportunity cost of not holding it.

Definition 19. Endogenous costs of liquidity are the result of the price impact of a trader's order.³⁶ In the analysis that follows and in the LARC application for assessing market liquidity risk, these costs primarily affect the systemic illiquidity of an issuer and are a primary signal of distress. An example of an asset with elevated endogenous costs due to increased risk is peripheral bonds of the Eurozone after the onset of the Eurozone Sovereign Debt Crisis. An example of the second kind of assets with increased endogenous costs are usually assets with safe haven status such as German government securities that are highly sought-after in times of market distress.

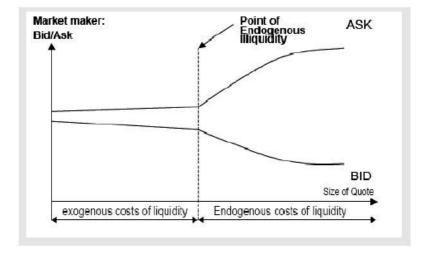
Exogenous and endogenous costs are presented in Figure 21³⁷ as Bid-Ask Spread vs. Size of Quote:

³⁴ Ibid., p. 10

³⁵ zeb/rolfes.schierenbeck.associates, Mag. Peter Madritsch, "Liquidity Risk – alternatives for quantification of liquidity risk", p. 19, pres. at Deutsche Bundesbank and Oesterreichische Nationalbank Advanced Seminar on Basel II, 2009

³⁶ Ibid., p. 19

³⁷ Ibid., p. 19



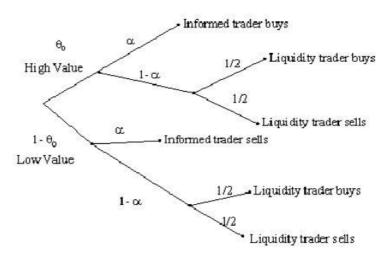
BOX 2: Market Microstructure concepts applied

Market Microstructure asserts that market participants are either liquidity demanders or liquidity suppliers (See BOX 1: Definitions of concepts underlying the model). As liquidity suppliers act on incomplete and imperfect information as they do not know if they are trading against an informed or an uninformed liquidity demander, they demand premium in the form of bid-ask spread.

The trading sequence is presented in Figure 22³⁸:

 $^{^{\}rm 38}$ Foucault, Thierry (HEC, Paris), "The Informational Content of Order Flow", 2007, p.8

TRADING SEQUENCE



Market Mictrostructure recognizes the role of:

Private Information – informational asymmetries among market participants are prevalent in securities markets.

Heterogeneity among Market Participants – participants in securities markets have various objectives (e.g. dealers are different from final investors; hedgers are different from speculators etc.).

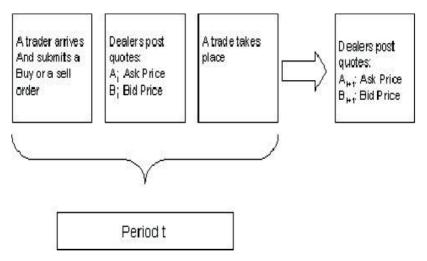
Institutional Framework - market design and market regulation matter.³⁹

A key element of Market Mictrostructure that is a fundamental assumption to the findings later presented in this paper is that trade is sequential (i.e. sequential trade models) and the timing of events in each period is presented in Figure 23^{40} :

³⁹ Foucault, Thierry (HEC, Paris), "Market Microstructure Approach", 2007, p. 6

 $^{^{\}rm 40}$ Foucault, Thierry (HEC, Paris), "The Informational Content of Order Flow", 2007, p.7

OCCURRENCE OF EVENTS WITHIN PERIOD⁴¹



^{*} Orders are all for 1 round lot of the risky security

Let μ be the fundamental price of a security, as determined by the CAPM for instance, V be the liquidation value of the security, V_H be a high liquidation value, V_L be a low liquidation value. Let V be a random realization of the price process that determines V and F be a filtration of the price process of the asset traded that gives a realization of value V-tilde.

Let α be defined as the probability of trading against an informed trader, and θ – as the probability of high liquidation value. Furthermore, let A be an ask quote, B be a bid quote, t be a given moment in time.

Liquidation Value of the security is uncertain and is revealed at the end of the trading day. It can be large (V_H) or small (V_L) with equal probabilities. $V_H - V_L = \sigma$.

In Foucault, Thierry (HEC, Paris), "The Informational Content of Order Flow", 2007 (pp. 11–15) it is shown that:

The dealer's expected profit conditional on executing a buy order in period t is:

$$\frac{\theta_{t-1}\alpha}{\theta_{t-1}\alpha + \frac{1-\alpha}{2}} (A_t - V_H) + \frac{1-\alpha}{2(\theta_{t-1}\alpha + \frac{1-\alpha}{2})} (A_t - \mu_{t-1})$$
 (1.8)

The dealer bears a loss when he trades with an informed trader and a proft when he trades with a liquidity trader if:

$$\mu_{t-1} < A_t < V_H \tag{1.9}$$

The expected-zero-profit ask price in period t is therefore:

$$A_{t} = \mu_{t-1} + \sigma \frac{2\alpha \theta_{t-1} (1 - \theta_{t-1})}{1 + \alpha (2\theta_{t-1} - 1)}$$
(1.10)

Same reasoning for the determination of the bid price in period t:

$$B_{t} = \mu_{t-1} - \sigma \frac{2\alpha \theta_{t-1} (1 - \theta_{t-1})}{1 + \alpha (1 - 2\theta_{t-1})}$$
(1.11)

Observe that $A_+ > B_+$ if $\alpha > 0$.

If $0 < \theta_{t-1} < 1$ and $\alpha > 0$ then there is a bid-ask spread:

$$B_t < \mu_{t-1} < A_t \tag{1.12}$$

Actually it can be shown that:

$$A_{t} = E(\widetilde{V} | F_{t-1}, x_{t} = +1) = \mu_{t}(+1)$$
(1.13)

$$B_{t} = E(\widetilde{V} | F_{t-1} x_{t} = -1) = \mu_{t}(-1)$$
(1.14)

where (+1) denotes a buy order and (-1) denotes a sell order, giving the sign of the transaction.

And it follows that if a buy order is submitted at date t, the dealer's beliefs for the value of the security at time t+1 are:

$$\theta_{t}(+1) = Prob(V = V_{H} | x_{t} = +1) = \frac{1 + \alpha}{1 + \alpha(2\theta_{t, t} - 1)} \theta_{t-1} > \theta_{t-1}$$
 (1.15)

Hence, dealer's beliefs and quotes are revised upward after receiving a buy order.

Respectively, if a sell order is submitted, the dealer's beliefs for the value of the security at time t + 1 are:

$$\theta_{t}(-1) = Prob(V = V_{H} | x_{t} = -1) = \frac{(1 - \alpha) \theta_{t-1}}{1 + \alpha(1 - 2\theta_{t-1})} < \theta_{t-1}$$
(1.16)

Hence, dealer's beliefs and quotes are revised downward after receiving a sell order.

Thus the sign of a trade is a signal that carries information with regard to the expected value of a security. On the other hand, as bid-ask spreads are a measure of market illiquidity, it follows that changes in market illiquidity

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would imply expectations for the value of a security. There are various studies in the field of return-illiquidity relations such as Amihud (2000).⁴¹

As noted, assertions are generally made for one round lot of the risky security. In the present analysis, the first augmentation that we make is that we assess the effective bid-ask spread, i.e. the dynamic of bid-ask spread depending on lot size. Further, we will look not only at the effective bid-offer spread but also at its interplay with maturity, another fundamental factor in fixed income and liquidity considerations.

BOX 3: Regulatory overview of liquidity risk measurement and management

The existence of liquidity risk directly jeopardizes the solvency of an institution if grossly mismanaged. Not surprisingly, it has been directly associated with market turmoil in recent years - from the credit crunch due to the tightening of wholesale funding post Lehman Brothers' failure through the freeze of the Covered Bonds Market to the thin market for bonds of the Eurozone Periphery, the latter two prompting action by the European Central Bank (ECB). Thus supervisors put more emphasis on the management of liquidity risk. ⁴²

Examples of regulatory effort in this respect in European and global perspective are: (i) the Committee of European Banking Supervisors (CEBS) Technical Advice to the European Commission (EC) on Liquidity Risk Management (June 2008); (ii) the BIS Principles for Sound Liquidity Risk Management and Supervison (Sept. 2008); (iii) the adaptations of the Basel II into the EC Directives 2006/48 and 2006/49 (Oct. 2008); (iv) the BIS Principles for Sound Stress Testing Practices and Supervision (Jan. 2009); (v) the CEBS Guidelines on Liquidity Buffers (Dec. 2009): and (vi) the CEBS Guidelines on Liquidity Cost Benefit Allocation (Oct.2010). These efforts culminated in the creation of the Third Basel Accord: International Framework for Liquidity Risk Measurement, Standards and Monitoring (Basel III, Dec. 2010). Based on the Basel III, the EC went further with the proposal to harmonize the existing liquidity management practices of the credit institutions in EU by 2015 (July 2011). As a first step a general requirement for banks to keep appropriate liquidity coverage from 2013 is envisaged. The Commission also proposes to keep the power of specifying further the liquidity coverage require-

⁴¹ Amihud, Yakov (Stern School of Business, New York University) "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects", 2000

⁴² Bhimalingam, Mahesh and Nick Burns, CFA, "Fundamental Credit Special: Basel III and Solvency II - Impact on credit markets" Deutsche Bank Special Report, 2011, p. 14

ment in line with conclusions from the observations period and international developments. The European Banking Authority was established by Regulation 1093/2010 of the European parliament and of the European Council. As of January 1, 2011 it has taken over all of existing and ongoing tasks and responsibilities from the CEBS.

For sake of clarity, some additional concepts are defined below in relation to liquidity ratios and the regulatory framework:

Definition 20. Available Stable Funding (ASF) – the total amount of an institution's ASF is equal to:

- · Capital (after deductions);
- Preferred stock with maturity ≥ 1 year;
- Liabilities with effective maturity ≥ 1 year;
- That portion of stable (namely sticky) deposits in a time of stress;
- That portion of wholesale funding of maturity \leq 1 year that can stay with the institution in a time of stress. ⁴³

Definition 21. Required Stable Funding (RSF) – the RSF is a function of the assets on the balance sheet of the bank. The RSF Factor (0-100%) applied to each asset type (on- and off- balance sheet exposures) approximates the amount of a particular asset that could not be monetised through sale or used as collateral in a secured borrowing on an extended basis during a liquidity event lasting one year. Encumbered assets on the balance sheet will get a 100% RSF, unless the encumbrance period remaining \leq 1 year. The definition of "encumbered assets" to include covered bonds and securitisations held on balance sheet could make current sources of funding inefficient.⁴³

In the following section we shall present the regulatory view on managing liquidity risk, some indicators based on the balance sheet of institutions and the concepts defined herein used for assessing liquidity risk. Then we shall consider a market microstructure perspective on the problem and venture into addressing the problem of assessment and control of liquidity risk in one of the more complex and, due to this, currently less popular but very powerful ways – market liquidity management through adjusting liquidity cushions, dependent on the issuer's idiosyncracies and conditional on the prevailing market conditions.

Traditional approaches to managing liquidity risk include the use of liquidity ratios/indicators and liquidity gap analysis, both based on balance sheet and cashflow modeling. Liquidity ratios are the easiest and one of the

⁴³ Ibid., p. 15

most widespread ways of assessing the liquidity adequacy of an institution. Examples include ⁴⁴:

$$Liquidity Index = \frac{Maturity-Weighted Assets}{Maturity-Weighted Liquilities}$$

Asset Term Risk Ratio =
$$\frac{\text{Loans with Delayed Payments}}{\text{Total Loan Volume}}$$

$$\label{eq:Liquidity Reserves} \mbox{Liquidity Coefficient} = \frac{\mbox{Liquidity Reserves}}{\mbox{Short-Term Obligations}}$$

$$\mbox{Liquidity Outflow Risk} = \frac{\mbox{Open (Large) Credit Approvals}}{\mbox{Liquidity Reserves}}$$

Liquidity ratios and indicators in the Basel III framework include:

Net Stable (NSFR) =
$$\frac{\text{Available Stable Funding (ASF)}}{\text{Required Stable Funding (RSF)}}$$

This ratio has to be greater than 100%.⁴⁵

⁴⁴ zeb/rolfes.schierenbeck.associates, Dr. Lars Kleffmann, "Liquidity Risk – regulatory framework and liquidity cash flow modelling", pres. at Deutsche Bundesbank and Oesterreichische Nationalbank Advanced Seminar on Basel II, 2009

⁴⁵ Bhimalingam, Mahesh and Nick Burns, CFA, "Fundamental Credit Special: Basel III and Solvency II – Impact on credit markets" Deutsche Bank Special Report, 2011, p. 14

This ratio has to be greater than 100%⁴⁶
Net cash outflows = Outflows - min{inflows, 75% of outflows}⁴⁷
Overall, classic liquidity indicators⁴⁸ are based on and include:

Loan-to-Deposit Ratio where the volume of deposits should be higher than the volume of loans;

Golden Rule of Banking where the maturity structure of assets should be the same as the maturity structure of liabilities;

Bottomline Theory which assumes that a certain amount of short-term deposits remains for a long time in the bank and is therefore available for liquidity management;

Shiftability Theory which considers the possibility to sell assets ahead of time (before maturity) to get liquidity.

It is this latter prospect for which we propose an evaluation method in this paper. It is one of the ways in which gaps in the accumulated cashflows, which are in essence the institution's liquidity gaps, can be eliminated.

Methods of liquidity assessment such as Liquidity-Adjusted Value at Risk (LVaR), whereby exogenous liquidity costs are integrated in the VaR figure, and Liquidity at Risk (LaR), whereby the endogenous liquidity costs are assessed, currently exist. Thus, they each address one aspect of the cost at which the aforementioned shifting in maturity would take place, whereas the LARC method, despite its very high complexity and data requirements, addresses both aspects of that cost (endogenous and exogenous). As discussed, transaction costs to a large extent follow from the way markets function on a micro level, which is addressed in **BOX 2: Market Microstructure Concepts Applied.**

⁴⁶ Ibid., p. 15

⁴⁷ Ibid., p. 16

⁴⁸ zeb/rolfes.schierenbeck.associates, Mag. Peter Madritsch, "Liquidity Risk – alternatives for quantification of liquidity risk", p. 4, pres. at Deutsche Bundesbank and Oesterreichische Nationalbank Advanced Seminar on Basel II. 2009

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