A Supply-Side Model Under Currency Board Rules

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Abstract

This paper provides a dynamic model of non-orthodox currency board based on the monetary approach for a small open economy. A basic rule of the currency board system postulates, that changes in the monetary base of the country have to be equivalent to changes in its foreign reserves. The non-orthodox currency board admits some more flexibility and is currently used in some transition countries. The model is used for investigating how shocks, which are relevant for a transition economy could affect the real money balances and the real wages for drawing economic policy implications.

1. Basic currency board equations.

The essentials of the currency board arrangements are very much in line of the gold standard principles. The basic idea of golden standard is to ensure credibility of the currency, baking it by gold. In the case of a currency board system, the same principle applies baking the domestic currency by monetary assets of one or more hard currencies in order to transfer their higher credibility. To be more concrete, the monetary base of the local currency has to be baked by foreign hard currency reserves of the country.

It seems reasonable to apply a monetary approach for modeling a currency board system, because it basics seem to fit better to the basic statements of the monetary approach (Willms [12].):

1. The exchange rates are constant and correspond to purchasing power parity.
2. Prices and wages are flexible.
3. The economies are in a full employment balance.
4. Current account changes imply corresponding changes in the monetary base, i.e. there are no neutralization operations of the Central Bank.

5. Currency is the only financial asset. Local currency is hold by locals and foreign currency is hold by foreigners.

The modern monetary approach was developed mainly by Mundell (7), (8), Fleming (4), Johnson (6), Dornbusch (3)

Now, we turn to convert the principles of the non-orthodox currency board system and the basic statements of the monetary approach into mathematical terms and equations.

We suppose that the equations are dynamic, which means that all quantities involved, do depend on time, denoted by *t*.

A basic rule of the currency board requires the domestic monetary base *B* to be baked by the foreign reserves *F*:

$$B=F \tag{1}$$

From (1) follows that the absolute change dB of the monetary base has to be equal to the absolute dF of the foreign reserves:

$$dB = dF \tag{2}$$

The relative changes of these variables are equal as well:

$$b = \frac{dB}{B} = \frac{dF}{F} = f \tag{3}$$

Equation (1) with its versions (2) and (3) is a basic currency board equation. It is a prescription saying that the Central Bank has to change the monetary base in the same rate the foreign reserves have changed.

It is well known, that the money supply *M* is equal to the product of the money multiplayer μ and the monetary base *B*:

$$M = \mu B \tag{4}$$

Replacing the monetary base *B* from (1) into (4) we get the basic money supply under currency board equation:

$$M = \mu F \tag{5}$$

Taking the differentials of both sides of the last equation and deviding them by (5) we get:

$$dM = \mu dF + F d\mu \tag{6}$$

$$\frac{dM}{M} = \frac{d\mu}{\mu} + \frac{dF}{F} \tag{7}$$

Denoting $m = \frac{dM}{M}$, $f = \frac{dF}{F}$, $\varepsilon = \frac{d\mu}{\mu}$ we get

$$m = f + \varepsilon \tag{8}$$

Equation (8) shows that the relative change of the money supply is equal to the sum of the relative change of the foreign reserves and the money multiplier. The money multiplier is whethter constant or changes slowly with the time, so its absolute and relative change should be zero:

$$d\mu = 0, \quad (9)$$

$$\varepsilon = 0 \quad (10)$$

Replacing (9) in (6) and (10) in (8) we get

$$dM = \mu dF \quad (11)$$

$$m = f \quad (12)$$

The absolute change *dF* of the foreign reserves *F* of a country for the time period *dt* is indeed the balance of payments *BP* for the same period of time

$$dF = BPdt \quad (13)$$

Similarly, for the corresponing relative changes we would have:

$$df(t) = b_p(t)dt \quad (14)$$

with $b_p = \frac{dBP}{BP}$

Consequently

$$\dot{f} = b_p \quad (15)$$

From (12) and (15) we get

$$\dot{m} = b_p \quad (16)$$

The last equation is another basic currency board equation, showing that the rate of money supply relative change is equal to the relative change of balance of payments.

2. The model.

For constructing the model we will use relationships based on the *IS, LM, AD, BP* curves.

The IS relationship follows from the basic open economy identity

$$Y=C(Y)+I(i)+G+NX(Y,Y^*,EP/P^*), \quad (17)$$

with $NX=Ex\text{-}Im=CA$ being the net export or the current account. Considering G, Y^*, E as exogeniously fixed variables, we have a relationship between the interest rate i and the output Y, giving indeed the *IS* curve.

The LM relationship follows from the equality

$$\frac{M}{P} = L(Y,i) \quad (18)$$

of the real money supply (balances) and demand. Considering the real money balances exogeniously fixed, we get an interst rate – output relationship

$$i = \Phi(Y, \frac{M}{P}) \quad (19)$$

which is indeed the *LM* curve.

The *LM* curve could be transformed into the relative changes form in the following way:

$$m - p = \eta y - \sigma i \qquad (20)$$

where $m = \frac{dM}{M}, p = \frac{dP}{P}, y = \frac{dY}{Y}, \eta = \eta_{\frac{M}{P}(Y)} = \frac{\frac{d\frac{M}{P}}{\frac{M}{P}}}{\frac{dY}{Y}}, \sigma = \sigma_{\frac{M}{P}(i)} = -\frac{\frac{d\frac{M}{P}}{\frac{M}{P}}}{i}$

For the elastisities: $\eta > 0$, because $\frac{dL}{dY} > 0$; $\sigma > 0$, because $-\sigma = \frac{dL}{di} < 0$

For the interest we get

$$i = \frac{\eta}{\sigma} y - \frac{1}{\sigma}(m - p)$$

The AD curve

Replacing the interest *i* from equation (20) of the *LM* curve into equation (17) of the *IS* curve, we get:

$$Y = C(Y) + I(\Phi(Y, \frac{M}{P})) + G + NX(Y, Y^*, E) \qquad (21)$$

Solving the last equation withrespect to *Y* we get:

$$Y = AD(G, \frac{M}{P}, Y^*, E) \qquad (22)$$

Considering the external output Y^* and the exchange rate *E*, as exogeniously fixed variables we have the right-hand side of equation (22) as a function of the variables *G* and $\frac{M}{P}$:

$$Y = AD(G, \frac{M}{P}) \qquad (23)$$

The right-hand side of the last equation is indeed the aggregate demand for domestic output, we denote by Y^d:

$$Y^d = AD(G, \frac{M}{P}) \qquad (24)$$

The left-hand side *Y* of equation (23) is indeed the aggregate supply Y^s of domestic output. Thus

$$Y = Y^s = Y^d = AD(G, \frac{M}{P}) \qquad (25)$$

Equation (25) shows that (23) gives the equilibrium demand and supply of domestic output.

We transform equation (24) into the relative changes form:

$$y^d = \lambda g + \delta(m - p) \qquad (26)$$

where $y^d = \frac{dY^d}{Y^d}, g = \frac{dG}{G}, \lambda = \lambda_{AD(G)} = \frac{\frac{dAD}{AD}}{\frac{dG}{G}}, \delta = \delta_{AD(\frac{M}{P})} = \frac{\frac{dAD}{AD}}{\frac{d\frac{M}{P}}{\frac{M}{P}}}$

The aggregate demand curve BP

The balance of payments *BP* are of crusual importance in the currency board mechanism, because it is the way to change the monetary base by changing foreign reserves. It is well known that:

$$BP = CA(Y, Y^*, \frac{EP}{P^*}) + KA(i - i^*) \tag{27}$$

where the current account *CA* depends on the domestic output *Y*, on the external output Y^* and on the terms of trade $\frac{EP}{P^*}$, denoting by *E* – the exchange rate, *P* – the domestic price level, P^* the external price level. The capital account (net capital inflow) *KA* depends on the domestic interest rate *i* and the external one i^*.

Replacing in (1) the expression

$$C(Y) + I(i) + G = Y^d \tag{28}$$

we get

$$Y = Y^d + NX(Y, Y^*, \frac{EP}{P^*}) \tag{29}$$

Thus

$$\frac{Y}{Y^d} = 1 + \frac{1}{Y^d} NX(Y, Y^*, \frac{EP}{P^*}) \tag{30}$$

Resolving the last equation with respect to Y^* we express it as a function *f* of the other variables:

$$Y^* = f(\frac{Y}{Y^d}, \frac{EP}{P^*}) \tag{31}$$

Replacing (31) into (27) we express *BP* as a function *g* of the variables $\frac{Y}{Y^d}, \frac{EP}{P^*}, i - i^*$:

$$BP = g(\frac{Y}{Y^d}, \frac{EP}{P^*}, i - i^*) \tag{32}$$

Using the same mathematical technique we got (26) from (24) we derive now (33) from (32):

$$b_p = \alpha(y - y^d) + \beta(e + p - p^*) + \gamma(i - i^*) \tag{33}$$

denoting the corresponding relative changes: $b_p = \frac{dBP}{BP}$ $y = \frac{dY}{Y}$, $y^d = \frac{dY^d}{Y^d}$, $e = \frac{dE}{E}$, $p = \frac{dP}{P}$, $p = \frac{dP^*}{P^*}$, and α, β, γ stay for the elastisities:

$$\alpha = \alpha_{BP(\frac{Y}{Y^d})} = \frac{\frac{dBP}{BP}}{\frac{d\frac{Y}{Y^d}}{\frac{Y}{Y^d}}},\ \beta = \beta_{BP(\frac{EP}{P^*})} = \frac{\frac{dBP}{BP}}{\frac{d\frac{EP}{P^*}}{\frac{EP}{P^*}}},\ \gamma = \gamma_{BP(i-i)} = \frac{\frac{dBP}{BP}}{i-i^*}.$$

Replacing b_p from equation (33), in (17), we get

$$\dot{m} = \alpha(y - y^d) + \beta(e + p - p^*) + \gamma(i - i^*) \tag{34}$$

The rate of price level change being indeed the inflation is considered to be proportional to the gap between the demand Y^d and the supply Y of domestic output expressed by the relative changes

$$\dot{p} = \pi(y^d - y) \tag{35}$$

for π being positive

Subtracting (35) from (34) we get

$$\dot{m} - \dot{p} = (\alpha + \pi)(y - y^d) + \beta(e + p - p^*) + \gamma(i - i^*) \tag{36}$$

Replacing y^d from (26) into (36) we get:

$$\dot{m} - \dot{p} = (\alpha + \pi)(y - \lambda g - \delta(m - p)) + \beta(e + p - p^*) + \gamma(i - i^*) \tag{37}$$

For the real money balances we set

$$m - p = h$$

and replace it into (37):

$$\dot{h} = (\alpha + \pi)(y - \lambda g - \delta h) + \beta(e + p - p^*) + \gamma(i - i^*) \tag{38}$$

By elementary calculations:

$$\dot{h} = -(\alpha + \pi)\delta h + (\alpha + \pi)y - (\alpha + \pi)\lambda g + \beta(e + p - p^*) + \gamma(i - i^*) \tag{39}$$

Replacing y^d from (26) into (35) and $m - p = h$ we get

$$\dot{p} = \pi(\lambda g + \delta h - y)$$ and

$$\dot{p} = \pi\delta h + \pi\lambda g - \pi y) \tag{40}$$

Labour market equilibrium

The equilibrium of labour market is determined by the demand $L^{(d)}(\frac{W}{P})$ and supply $L^{(s)}(\frac{W}{P})$ of labour depending on the price of labour, i.e. the real wage $\frac{W}{P}$. The rate of change of the nominal wage is proportional to the difference between demand and supply of labour, which itself is propotional to the difference between relative changes of wages and prices:

$$\dot{w} = \varepsilon(l^{(d)}(w - p) - l^{(s)}(w - p))$$

$$l^{(d)}(w-p)-l^{(s)}(w-p)=\chi(w-p)$$
$$\dot{w}=\varepsilon\chi(w-p)$$
Setting $\psi=\varepsilon\chi$ in the last equation, we get
$$\dot{w}=\psi(w-p) \qquad (41)$$
Subtracting (40) from (41), we get
$$\dot{w}-\dot{p}=\psi(w-p)-\pi\delta h-\pi\lambda g+\pi y$$
In the last equation we set $w-p=x$:
$$\dot{x}=\psi x-\pi\delta h-\pi\lambda g+\pi y \qquad (42)$$
Equations (39) и (42) could be written in matrix form:
$$\begin{bmatrix}\dot{h}\\ \dot{x}\end{bmatrix}=\begin{bmatrix}-(\alpha+\pi)\delta & 0\\ -\pi\delta & \psi\end{bmatrix}\begin{bmatrix}h\\ x\end{bmatrix}+\begin{bmatrix}(\alpha+\pi)y-(\alpha+\pi)\lambda g+\beta(e+p-p^*)+\gamma(i-i^*)\\ \pi y-\pi\lambda g\end{bmatrix} \qquad (43)$$

3. Dynamic stability of the system

The dynamic system stability conditions are:

$Det=-(\alpha+\pi)\delta\psi<0$ because $\alpha>0, \pi>0, \delta>0, \psi>0$

In this case, the dynamic system is inherently unstable and it has a stable solution, only if there is a saddle path solution. Such a solution, however, requires a free variable that is able to jump discretely to put the model on a unique stable path. Given that money, output and prices are all slow-moving variables, the only candidate for a jump variable is the money-wage rate. By assuming that the money-wage rate can jump at a moment in time - such as when a new set of wage contracts are signed - the model can give a stable solution.

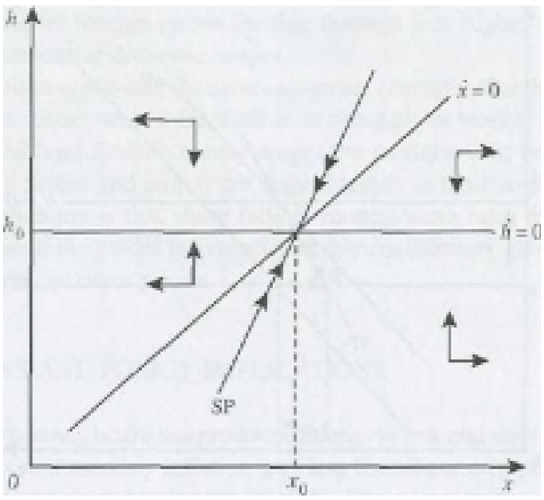

Figure1. The stability in the model

This is shown in Figure 1 where the $\dot{h} = 0$ locus is horizontal since real money balances are independent of the real-wage rate. The real wage equilibrium schedule, on the other hand, has a positive slope since a higher level of real money balances are associated with higher output, through a trade balance surplus, and the higher production leads to higher real wages through labour market pressure. Figure 1 gives the direction arrows, showing the model to be unstable unless on the stable arm denoted by SP.

4. Economc policy implications.

We can now analyse shocks in the system.

A cut in government spending (or a rise in the long-run level of output) will raise both the money-market equilibrium line and the constant real-wage line. The new equilibrium will be reached by an initial downward jump in the money-wage rate, since rational agents will anticipate a fall in public spending (or rise in output) to reduce prices. This will stimulate the demand for output and raise the real money supply through a balance of trade surplus. Note in Figure 2., that initially the real-wage rate falls below its long-run level - that is, it overshoots the final equilibrium at *C* moving from point *A* to point *B* - due to the stickiness of prices and output in the model. It is also important to note that a cut in aggregate demand had the effect of raising output in this model because of its effect on the supply-side through the money wage. This is potentially important for Bulgaria, since reducing state spending without liberalizing the supply-side will not lead to output growth.

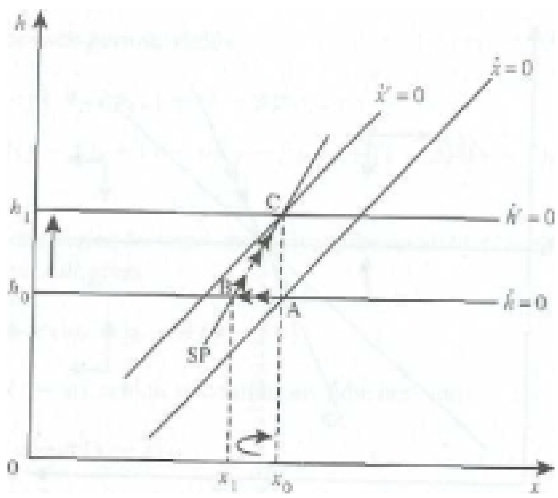

Figure 2. The effect of a rise in output

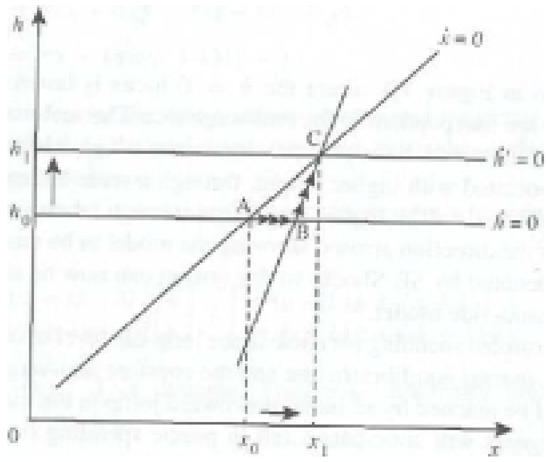

Figure 3. A trade balance shock

An exogenous trade balance shock - such as a rise in the foreign price level -leaves the constant real-wage schedule unaffected, but causes the real-money balance locus to shift up. Figure 3. shows that the real-wage rate initially jumps, from point *A* to point *B*, but undershoots its final equilibrium level at x_1. The higher foreign price level initially raises the money supply which in turn stimulates demand and domestic prices. The money wage jumps because wage-setters anticipate the higher foreign prices feeding through into higher domestic prices and a higher demand for domestic output.

Important remark: The model assumes, crucially, that the labour market always clears as money wages are flexible to bring about market clearing at each point in time. Without flexible money wages the model would become unstable, as goods market prices and output are only sluggish in their response to shocks. The policy implication is, that sharp falls in money-wage rates may be essential from time to time, if the model is to reach a stable equilibrium, given the degree of stickiness apparent in other prices

References

1. Bulgarian National Bank Law. State Newspaper Nr. 46, June 10, 1997. (Bulgarian).
2. Commercial Banks Law. State Newspaper Nr. 52, July 1, 1997. (in Bulgarian)
3. Rudiger Dornbush. Expectations and Exchange Rate Dynamics. Journal of Political Economy, vol. 84, no. 6, December 1976.
4. Marcus J. Fleming. Domestic Financial Policies under Fixed and Flexible Exchange Rate. IMF Staff Papers, 9, November 1962, p. 369-79.
5. Milton Friedman. Essays in Positive Economics. The University of Chicago Press, Chicago, 1953.
6. Harry G. Johnson. The Monetary Approach to Balance-of-Payments Theory. In: Johnson, H. G. (Ed.) Further Essays in Monetary Economics, London, 1972, p. 229-249.
7. Robert A. Mundell. Capital mobility and stabilization policy under fixed and flexible exchange rates. Canadian Journal for Economics and Political Science, 29, November 1963, p. 475-85.
8. Robert A. Mundell. A reply: Capital mobility and size. Canadian Journal for Economics and Political Science, 30, August 1964, p.421-31.
9. Maurice Obstfield and Kenneth Rogoff Foundations of International Macroeconomics. The MIT Press, 1996.
10. Kent Osband, Delano Villanueva. Independent Currency Authorities: An Analytic Primer. IMF Staff Papers, Vol. 40, No. 1, March 1993, pp. 202-216.
11. George Chobanov, Eric J. Pentecost. Price and Output Dynamics under Currency Board with an Application to Bulgaria. In "Exchange Rate Policies, Prices and Supply-Side Response. A Study of Transitional Economies", Eds. Christos Papazoglou and Eric Pentecost, Palgrave, 2001, pp. 91-103.
12. Manfred Willms. Intenationale Waehrungspolitik. Verlag Franz Vahlen, Muenchen, 1992.
13. Ben J. Heijdra, Frederick van der Ploeg. The Foundations of Modern Macroeconomics. Oxford University Press, 2002.