# MODELLING AND FORECASTING THE VOLATILITY OF THIN EMERGING STOCK MARKETS: THE CASE OF BULGARIA

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## Abstract

*Modern Portfolio Theory associates the stock market risk with volatility of the return. Volatility is measured by the variance of return but the investment community does not accepted this measure, since it weighs equally the deviations of the average return, while most investors determine the risk on the basis of small or negative returns. In the last few years the measure Value at Risk (VaR) has established itself in the practice.*

*The issue about modelling and forecasting thin emerging stock markets risk is still open. The subject of the paper is the risk of the Bulgarian stock market. The aim of the paper is to give the investment community a model for assessment and forecasting of the Bulgarian stock market risk.*

*The results of the research show that the SOFIX index has basic characteristics of most of the emerging stock markets, namely: high risk, significant autocorrelation, non-normality, volatility clustering. Three models have been applied – RiskMetrics, EWMA with t distributed innovations and EWMA with GED distributed innovations. The EWMA with t distributed innovations and EWMA with GED distributed innovations adequately evaluate the risk of the Bulgarian stock market.*

**Key words**: Bulgarian stock market, volatility, EWMA, Value-at-Risk

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Modern Portfolio Theory associates the stock market risk with volatility of the return. Volatility is measured by the variance of return but the investment community does not accepted this measure, since it weighs equally the deviations of the average return, while most investors determine the risk on the basis of small or negative returns.

In the last decade the measure Value at Risk (VaR) has established itself in the practice. It is part of the New Basel Accord. Value-at-Risk is defined as the maximum expected loss for a given horizon in specified confidence level.

The issue about modelling and forecasting thin emerging stock markets risk is still open. The aim of the paper is to give the investment community a model for assessment and forecasting of the Bulgarian stock market risk.

The paper is structured as follows: Literature review is presented in Part 1. Data and methodology are outlined in Part 2. Empirical results are presented in Part 3. Conclusion is given in Part 4.

## 1. Literature Review

There are numerous investigations on emerging equity markets that outline the different characteristics of emerging equity markets. Bekaert et.al. (1998) argued that emerging markets are highly non-normal. Moreover, seventeen of twenty stock markets exhibit positive skewness in the returns, and nineteen of twenty are leptokurtic over the investigated period April 1987-March 1997. Furthermore, there is no strong evidence that the non-normality found in many emerging market returns became less prominent in the 1990s.

Bekaert et.al. (1998) pointed out that correlation varies depending on both the state of economy and the state of the equity markets in each country. Correlation is higher in recessions and lower in recovers compared to the average correlation during both economic states. Moreover, the same asymmetry of correlation is observed in bear and bull markets: in bear markets correlation coefficients are higher while in bull markets correlation coefficients are lower.

Harvey (1995a) explained the high volatility of returns by (1) lack of diversification in the country index, (2) high risk exposures to volatile economic factors, and (3) time-variation in the risk exposures and/or incomplete integration into world capital market.

Harvey (1995b) found that the serial correlation in emerging markets returns is much higher than that observed in developed markets. He explained this feature as a lack of diversification and with the fact that trading depth induces spurious serial correlation. There are emerging markets partially integrated into the world capital market. Factors that contribute to market integration are free access by foreigners to domestic capital markets and free access by domestic investors to international capital markets. Potential barriers to integration come in the form of: access, taxes, and information.

In recent years modelling the time-varying nature of the volatility of emerging stock markets has attracted the interest of researchers. Aggarwal, Inclan, and Leal (1999) examined the stock market volatility of 10 largest emerging markets in Asia and Latin America. They found that shifts in volatility of considered emerging markets is related to important country-specific political, social, and economic events. Moreover, the time-varying stock market volatility is modelled by GARCH models.

Balaban, Bayar, and Faff (2003) forecasted stock market volatility of fourteen stock markets. They employed eleven models and use symmetric and asymmetric loss functions to evaluate the performance of these models. According to symmetric loss functions it is the exponential smoothing model that provides the best forecast. However, when asymmetric loss functions are applied ARCH-type models provide the best forecast.

Recently, several authors have investigated the volatility of Central and Eastern European stock markets. Kasch-Haroutounian and Price (2001), Glimore and McManus (2001), Poshakwale and Murinde (2001) and Murinde and Poshakwale (2002) found that significant autocorrelation, high volatility persistence, significant asymmetry, lack of relationship between stock market volatility and expected return and non-normality of the return distribution are basic characteristics of stock market volatility in transition countries.

We detect a gap in the literature that explains basic characteristics of stock market volatility in transition countries. We contribute to the literature by considering the Bulgarian stock market risk. We employ various models for forecasting of the Bulgarian stock market risk, measured as Value-at-Risk.

## 2. Methodology

Supervision institutions like BIS and IOSCO introduced Value-at-Risk (hereinafter, VaR) as a measure of the market risk in financial institutions. Jorion (1996, p.86) defined VaR as "the expected maximum loss (or worst loss) over a target horizon within a given confidence interval". In practice, three approaches are established for estimation of VaR: parametric (or variance-covariance), historical simulation, and Monte Carlo simulation. According to a survey of Deloitte and Touche Tohmatsu (2002) the most widely used approach is the parametric, popularized by the JP Morgan (1996). The parametric approach is easy to implement and computations are quickly. The main problem is related to VaR estimation of non-linear instruments (e.g. options).

The expected VaR estimate over the period t+1 could be calculated as follows:

$$VaR = \left|\delta(\alpha)\right| \times \sigma_{t+1} \tag{1}$$

where $\delta(\alpha)$ is $\alpha$-quintile of the standardized distribution, $\sigma_{t+1}$ is the standard deviation of $r_{t+1}$ conditional on the time t information set.

In recent years, researchers have widely used GARCH models for forecasting the stock market volatility, $\sigma_{t+1}$, in spite of the fact that the exponentially weighted moving average (hereinafter, EWMA) is the most popular model for stock market volatility forecasting among practitioners (Deloitte and Touche Tohmatsu, 2002). Dimson and Marsh (1990) gave another explanation of the popularity of the EWMA model. It states that sometimes sophisticated models could provide worse forecasts than naïve models. The main advantage of the EWMA is the simplicity of the calculating procedure with small number of available observations. The determination of the smoothing constant $\lambda$ is the most critical issue. Once the smoothing constant is determined one need two values to forecast the volatility: volatility at time t, $\sigma_t^2$ and squared return at time t, $r_t^2$. JP Morgan's *RiskMetrics* model is a variety of EWMA. They set $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data. The mathematical specification of it is as follows:

$$\sigma_{t+1}^2 = (1-\lambda)r_t^2 + \lambda\sigma_t^2 \tag{2}$$

The exponentially weighted moving averages model is a special case of the GARCH (1,1) model. Bollerslev (1986) proposed GARCH models for modelling volatility clustering and hence the leptokurtosis that is induced in the unconditional distribution of stock returns. In fact, in the presence of small number of observations the parameter estimates of the GARCH model are inefficient.

On the basis of numerous empirical studies on emerging stock markets we expect that the Bulgarian stock market will possess the typical characteristics of an emerging stock market: volatility clustering and leptokurtosis. The insufficient data set for GARCH modelling imposes application of EWMA model for modelling and forecasting Bulgarian stock market volatility. Following Guermat and Harris (2002) we estimate the vector of parameters of the EWMA model, $\theta$, using the Maximum Likelihood Method. The Log-likelihood function (Equation 3) is maximized by Marquardt's optimisation algorithm.

$$L\left(\theta = \sigma_{t+1}^2; \lambda; v_{t+1}; r_{-\infty}, .., r_t\right) = \sum_{s=0}^{\infty} (1-\lambda)\lambda^s \ln f\left(r_{t-s}; \sigma_{t+1}; v_{t+1}\right) \tag{3}$$

where $\lambda$ is smoothing constant, and $f\left(r_{t-s}; \sigma_{t+1}; v_{t+1}\right)$ is density function.

The leptokurtosis induced in unconditional distribution of stock market returns is captured with two models - EWMA-t and EWMA-ged. The first model assumes that stock market returns are Student –t distributed where the log-likelihood function of t-distribution has the following specification:

$$L\left(\theta = \sigma_{t+1}^2; \lambda; v_{t+1}; r_{-\infty}, .., r_t\right) = \sum_{s=0}^{\infty} (1-\lambda)\lambda^s \ln f\left(r_{t-s}; \sigma_{t+1}; v_{t+1}\right)$$
$$f_v(\theta) = \Gamma\left(\frac{v_{t+1}+1}{2}\right)\Gamma\left(\frac{v_{t+1}}{2}\right)^{-1}\left(\left(v_{t+1}-2\right)\sigma_{t+1}^2\right)^{-1/2} x\left(1 + r_{t+1}^2\left(\sigma_{t+1}^2\left(v_{t+1}-2\right)\right)^{-1}\right)^{-(v_{t+1}+1)/2}$$

4(4)

where $\Gamma$ is the Gamma function and $v$ represents the degrees of freedom.

Nelson (1995) proposed the Generalised error distribution for modelling fat-tailed nature of stock returns. The log-likelihood function is as follows:

$$L\left(\theta = \sigma_{t+1}^2;\lambda;v_{t+1};r_{-\infty},..,r_t\right) = \sum_{s=0}^{\infty}(1-\lambda)\lambda^s \ln f\left(r_{t-s};\sigma_{t+1};v_{t+1}\right)$$

$$f\left[r_t(\theta);v_{t+1}\right] = v_{t+1}\lambda^{-1}2^{-(1+1/v_{t+1})}\Gamma\left(\eta^{-1}\right)^{-1}\exp\left[-0.5\left|r_{t+1}(\theta)\lambda^{-1}\right|^{v_{t+1}}\right] \tag{5}$$

$$\lambda = \left[2^{(-2/v_{t+1})}\Gamma\left(v_{t+1}{}^{-1}\right)\Gamma\left(3v_{t+1}{}^{-1}\right)^{-1}\right]^{1/2}$$

where is the Gamma function and $v$ is the degrees of freedom.

The one-day ahead volatility, $\sigma_{t+1}$, is forecasted with three models: *RiskMetrics*, EWMA-t, and EWMA-ged. Forecasted volatility is replaced in Equation 1 to estimate Vale-at-Risk. VaR estimates are calculated at three different confidence intervals: 95%, 97.50%, and 99%. We employ Kupiec's (1995) methodology for evaluation of the adequacy of a particular model for VaR estimation. The null-hypothesis is that the number of violations follows a binomial distribution. The Kupiec's Likelihood Ratio is $\chi^2$ distributed with one degree of freedom and has the following specification:

$$LR = 2\ln\left(\alpha^*(1-\alpha^*)^{T-x}\right) - 2\ln\left(\alpha^x \times \beta^{T-x}\right), \tag{6}$$

where $\alpha^* = x/T$, x is number of violations from VaR estimates, T is the total number of VaR estimates, α is required risk level (5%, 2.50% and 1%), $\beta = 1-\alpha$. The null-hypothesis cannot be rejected at a confidence interval *p* if the percentage of violations of the VaR is equal to *α*, or if:

$$\alpha - \sqrt{\alpha(1-\alpha)\frac{\chi_p^2(1)}{T_t}} < \alpha^* < \alpha + \sqrt{\alpha(1-\alpha)\frac{\chi_p^2(1)}{T_t}} \tag{7}$$

where $T_t$ is the total number of VaR estimates, $\chi_p^2(1)$ is the critical value of the chi-squared distribution with one degree of freedom at a probability level p. When the percentage of violations is below the lower bound then the VaR overestimates the risk, when the percentage of violations is above the upper bound the VaR underestimates the risk.

### 3. Empirical Results

We consider the official stock market index of Bulgarian Stock Exchange – Sofia. We examine the daily returns of the SOFIX index over the period 24 October 2000 – 19 November 2004. The data is provided by Bulgarian Stock Exchange.

Figure 1 presents index values (right scale) and index returns (left scale) over the considered period. We observe downward trend over the period November 2000 – November 2001 and high volatility. The trend is upward after November 2001 and the volatility is lower.

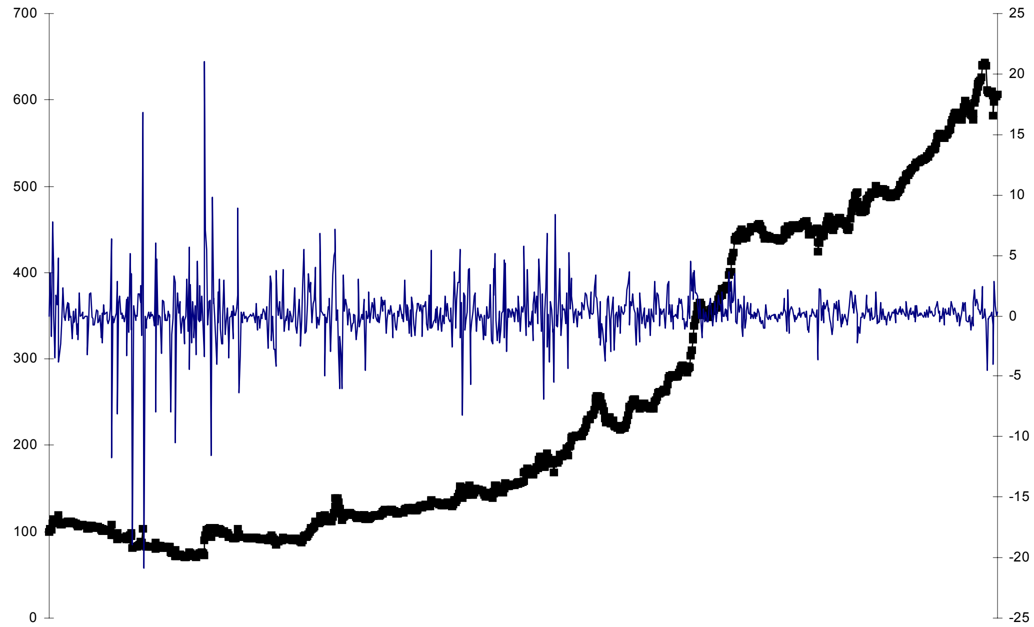


**Figure 1.** Daily values of SOFIX index

Tables 1 and 2 help us to derive basic distributional characteristics of Bulgarian stock market, represented by SOFIX index. We document high and positive average daily return 0.2034% (44.42% in annual basis) over the considered period. The stock market has high risk, measured by standard deviation of return. It is equal to 35.25% on the annual basis. The Jarque-Berra statistics is significant, which leads to the rejection of the null hypothesis of normality. The kurtosis is high and significant while the skewness is negative. Thus, negative extreme returns are more likely to occur than the normal distribution forecasts. The leptokurtosis in return distribution could be caused by volatility clustering.

**Table 1.** Descriptive statistics

| | |
|---|---|
| Mean | 0.1777% |
| Median | 0.1104% |
| Maximum | 21.0733% |
| Minimum | -20.8995% |
| Standard deviation | 2.2295% |
| Skewness | -0.4566 |
| Kurtosis | 29.4941* |
| Jarque-Berra | 29692* |

*Note: * denotes significance at 5% risk level*

The autocorrelation coefficients of stock market returns and squared stock market returns are presented in Table 2. We can observe significant autocorrelation coefficients. The significant autocorrelation in squared returns series proves the presence of volatility clustering that could be caused by high kurtosis values. Our findings of significant autocorrelation in returns series are consistent with Harvey (1995a). This could be caused by nonsynhronious trading.

**Table 2.** Autocorrelation coefficients of stock market return and squared stock market return

| | **Lag** | **1** | **2** | **6** | **12** | **24** |
|---|---|---|---|---|---|---|
| r | Autocorrelation coefficient | -0.1190* | 0.0580* | 0.0360* | 0.0550* | -0.0560* |
| | *Ljung-Box* Q-statistics | 14.5190 | 17.9390 | 25.2090 | 43.1130 | 64.5540 |
| $r^2$ | Autocorrelation coefficient | 0.2490* | 0.0120* | 0.0250* | 0.2370* | 0.0090 |
| | *Ljung-Box* Q-statistics | 63.2870 | 63.4400 | 64.5510 | 150.7700 | 163.3400 |

*Note: * denotes significance at 5% risk level*

The above drawn characteristics of SOFIX index coincide with those typical for emerging stock markets documented by Harvey (1995a) and Bekaert et. al. (1998). Bulgarian stock market has high risk, non-normal distributed returns, and significant autocorrelation that is caused by nonsynhronious trading.

Results from application of GARCH models meet our expectations that the parameter estimates will be inefficient and insignificant due to small number of observations. Results are not published because of space limitations and are available upon request. Thus, we continue calculating the smoothing constant of EWMA-t and EWMA-ged models.

Table 3 shows parameter estimates of both models. The smoothing constants are significant at 5% risk level. The EWMA-ged model has higher smoothing constant than the EWMA-t model. Thus, the past values of the volatility have grater impact on current volatility. Degrees of freedom are significant. This proves the “fat-tailed” distribution of the stock market return.

**Table 3.** Parameters estimates

| | **EWMA-t** | **EWMA-ged** |
|---|---|---|
| $\lambda$ | 0.8947* | 0.9134* |
| t-statistic | 89.9658 | 96.8492 |
| $\nu$ | 3.2582* | 0.7922* |
| t-statistic | 20.2908 | 23.0236 |
| Log likelihood | -1816.9670 | -1795.4440 |

*Note: * denotes significance at 5% risk level*

We employ the approaches of Guermat and Harris (2002) and Bams and Wielhouwer (2001). First we forecast the stock market volatility. Then we calculate the VaR for one-day horizon at three different confidence intervals. The evaluation results from VaR calculation are presented in Table 4. It reports the number of violations, Kupiec's Likelihood Ratio, and upper and lower bounds.

**Table 4.** Evaluation Results

| ***95% Confidence interval*** | | | |
|---|---|---|---|
| | ***EWMA-t*** | ***EWMA-ged*** | ***RiskMetrics*** |
| x | 16 | 43 | 33 |
| α* | 1.58% | 4.25% | 3.26% |
| α | 5.00% | 5.00% | 5.00% |
| Kupiec's LR | **33.59** | 1.26 | **7.31** |
| Upper bound | 6.34% | 6.34% | 6.34% |
| Lower bound | 3.66% | 3.66% | 3.66% |
| ***97.50% Confidence interval*** | | | |
| x | 9 | 23 | 27 |
| α* | 0.89% | 2.27% | 2.67% |
| α | 2.50% | 2.50% | 2.50% |
| Kupiec's LR | **14.26** | 0.22 | 0.11 |
| Upper bound | 3.59% | 3.59% | 3.59% |
| Lower bound | 1.41% | 1.41% | 1.41% |
| ***99% Confidence interval*** | | | |
| x | 4 | 9 | 12 |
| α* | 0.40% | 0.89% | 1.19% |
| α | 1.00% | 1.00% | 1.00% |
| Kupiec's LR | 4.85 | 0.13 | 0.33 |
| Upper bound | 1.79% | 1.79% | 1.79% |
| Lower bound | 0.21% | 0.21% | 0.21% |

*Note: Figures in bold face denotes that VaR estimates of particular model exceed form upper and lower bounds.*

Both EWMA-t *RiskMetrics* models overestimate the risk at 95% confidence interval. The percentage of violations is below the lower bound. The EWMA-ged model evaluates the risk adequately. Violations are within bounds. The EWMA-t also overestimates the risk at 97.50% confidence interval. *RiskMetrics* and EWMA-ged evaluate adequately Bulgarian stock market risk. The EWMA-ged has lower violations than *RiskMetrics* model. All models considered in this study adequately evaluate the risk of the Bulgarian stock market at 99% confidence interval. The EWMA-t model has the lowest number of violations.

**4. Conclusions**

The study examines the risk of the Bulgarian stock market, measured by Value-at-Risk. SOFIX index possess the basic characteristics of a typical emerging stock markets. On the basis of results obtained in the study we reach a number of important conclusions.

First. Exponentially weighting moving average models allow modeling and forecasting both time-varying volatility and kurtosis in returns. The model could be applied in thin stock markets with insufficient number of observations.

Second. The assumed probability distribution should be able to reflect the fat tailed behaviour of stock market returns and we propose Student –t distribution and Generalised error distribution as the most appropriate.

Third. The EWMA-ged model adequately evaluate the risk of Bulgarian stock market at both 95% and 97.50% confidence intervals. We propose the EWMA-ged since it allows capturing fat-tailed distribution of stock market returns.

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