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Send your comments and opinions to:
Publications Division
Bulgarian National Bank
1. Alexander Battenberg Square
1000 Sofia. Bulgaria
Tel.: (+359 2) 9145 1351, 9145 1978, 981 1391
Fax: (+359 2) 980 2425
e-mail: Dimova.L@bnbank.org
Website: www.bnb.bg

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Summary*. The paper develops a small-scale dynamic stochastic general equilibrium model (DSGE) of the Bulgarian economy along the lines of the New Open Economy Macroeconomics. Our small open economy produces traded and non-traded goods and both sectors face hybrid inflation dynamics and capital adjustment costs. We incorporate the currency board mechanism and cover the balance of payments explicitly. The law of one price holds in the tradable sector implying that under flexible output prices, the terms of trade adjust in a way such that the external relative price of tradables remains constant.

Policy simulations are used to investigate the impact of changes in the economic environment to which transition economies and Bulgaria in particular are currently exposed to. We explore the effects of a Balassa-Samuelson (BS) type shock for sectoral and overall inflation and discuss current account sustainability and implications for the internal real exchange rate. We find that under the currency board the BS effect is not as persistent as found for accession countries under a fixed exchange rate in related research. Inflationary pressures arising from real-catch up processes seem therefore not to prevent compliance with the Maastricht inflation criterion.

We further analyse the impact of lump-sum government expenditures on the trade balance and the current account. Fiscal policies under the currency board put upward pressures on the sector-specific inflation rates and tend to yield short-run trade balance surpluses. However, intertemporal wealth effects dominate causing overall current account deficits and losses in foreign reserves. The model is solved using Uhlig's toolkit.

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1 Introduction

The political integration of Bulgaria into the European Union is scheduled for January 2007 and the road to adopting the Euro is on the horizon. The restructuring of the Bulgarian economy from planned to market-based is still incomplete but recent economic developments are promising. Bulgaria has fared fairly well since installment of the currency board in 1997 which has helped to attain low inflation and macroeconomic stability. Inflation has declined from over 1000 percent to less than 5 percent today and real economic growth has been robust since then ranging between 4 to 6 percent.[1] Prudent fiscal and debt management policies in recent times ensure that the balance of payments is sustainable in the future and bring down external debt.

Whereas macroeconomic management is well on track in Bulgaria there is an often voiced concern for Euro-accession countries under fixed or heavily managed exchange rates that inflationary supply-side pressures might prevent soon compliance with the inflation criterion and delay ERM II entry. Under the automatic currency board the choice of stabilisation policies is limited to the fiscal stance and contractionary policies might be needed in order to cope with these pressures. On the other hand, the ongoing restructuring of the Bulgarian economy towards a market-based economy will still need expansionary fiscal policies directed towards infrastructure investments and the like that put demand-side pressures on the inflation rate.

The framework proposed here illustrates these upward pressures on domestic consumer price inflation (cpi) by both supply and demand side phenomena within the framework of the New Open Economy Macroeconomics. Our small open economy produces traded and non-traded goods and features sector specific hybrid inflation dynamics as well as capital adjustment costs. In particular, we incorporate the currency board mechanism and cover the balance of payments explicitly. We also investigate the economic consequences for the real exchange rate whether government spending falls on home-tradables or home services. For the ease of exposition the law of one price holds in the tradables goods production, in other words the foreign (EMU) firms cannot price-discriminate across the border. This assumption is heroic given the latest estimations of exchange rate pass-through in Bulgaria (see Dimitrova (2006), forthcoming). The exchange rate pass-through to overall inflation is found to be around 0.3 and the second stage of the pass-through effect defined as the elasticity of domestic prices to international prices is 0.27. The assumption of full pass-through from foreign tradables inflation to the domestic one can still be justified by the role of our model serving as a benchmark framework.

We proceed as follows. Section 2 discusses the related literature and 3 sets out the macroeconomic framework along the lines of the New Open Economy Macroeconomics (NOEM). Section 4 analyses the long-run equilibrium of the economy and section 5 describes the calibration to the Bulgarian data. Under 6 we provide responses of economic key variables following shocks in productivity and fiscal spending. Conclusions follow.

2 A New Open Economy Macroeconomics framework

Under the automatic currency board mechanism, the domestic base money is fully backed by the stock of foreign reserve holdings which removes any discretionary monetary policy power from the central bank. The mechanism establishes a direct link between the balance of payments evolvements and the domestic money supply. A balance of payments deficit causes automatic contraction in reserve money and therefore contracts domestic credit and vice versa. Modelling the currency board mechanism in general equilibrium models has so far not found widespread attention and models adapted to the Bulgarian case are rarely available. A general equilibrium model of the trade balance dynamics in Bulgaria is developed in Valev (2005) within a neoclassical growth model and hence abstracting from nominal rigidities. Desquilbet and Nenovsky

[1] See Sorsa (2002) for an overview of recent economic developments in Bulgaria.

(2003) discuss the stability of the currency board in the conditions of self-fulfilling exchange rate crisis within a money in utility framework. The latter model does not feature different production sectors and the authors acknowledge that inclusion of the latter would seriously affect inflation and real exchange rate dynamics.

We construct a model which is richer in structure than the above mentioned models and we follow the NOEM literature in methodology. The NOEM comprises macroeconomic frameworks with explicit microfoundations that incorporate some sort of price and/or wage rigidity which causes prices and monetary developments to influence the real economic environment. The basic structure of the model is closest to Natalucci and Ravenna (2003). Henriksson (2005), Benigno and Thoenissen (2003), as well as Bokil (2005) are other main influences. Whereas Natalucci and Ravenna (2003) only allow for forward-looking inflation dynamics we also allow for backward looking price setting which altogether generates hybrid, or "humped-shaped" impulse-responses of inflation. Hybrid inflationary performance can be thought of as a proxy of the slow restructuring of firms within the Bulgarian economy. Our model takes into account that for a suffi cient studying of business cycle dynamics in transition economies, investment and capital have to be incorporated in the model.

We integrate the currency board mechanism and model the balance of payments explicitly. We therefore obtain a direct link between the stock of foreign reserves holdings and domestic base money supply. To highlight the character of this contribution as a first step of modelling the currency board within the NOEM we refrain from any features that break the automation in the currency board mechanism. Our central bank has no possibility of influencing the domestic base money by any means. There is no monetisation of government debt, as the fiscal stance has to be in balance every period.

Bulgaria's currency board actually features non-automatic elements that cut the direct link between the balance of payments and domestic money supply (see Miller (1999) for the empirical evidence). Namely, there exists the requirement of commercial banks to maintain reserves in the central bank. Further, Nenovsky and Hristov (2002) argue that the inclusion of government fiscal reserves in the liability side of a currency board (i.e. covering them with international reserves) creates a discretionary channel of monetary policy transmission in Bulgaria. Government revenue and expenditure policies therefore directly impact the reserve money and, hence, the money supply. The model developed so far can therefore serve as the benchmark framework against which to judge outcomes of discretionary monetary policy possibilities under the Bulgarian currency board.

3 The model: building blocks

3.1 Households

3.1.1 Preferences and Decision Problem

The small open economy is inhabited by a continuum of households that reside on the interval $[0, 1]$. In any period, a household j receives utility from consuming the consumption index C_s^j which is a composite of non-traded goods consumption $C_s^{N,j}$ and tradable goods consumption $C_s^{T,j}$. $C_s^{T,j}$ consists of home produced tradable goods consumption $C_{H,t}^j$ and foreign produced tradable goods consumption $C_{F,t}^j$. The household derives utility from liquidity services provided by holding real money balances $\frac{M_s^j}{Ps}$. Utility from holding money balances is not covered by bond holdings which provide indirect utility through the income they generate. The household has disutility $-V(L_s^j)$ by giving up leisure for supplying labour to firms in the tradable sector T

and non-tradable sector N. Each household maximises the following life-time utility function

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{(s-t)} [U(C_s^j) + N(\frac{M_s^j}{P_s}) - V(L_s^j)] \tag{1}$$

which is additively separable in the per-period functions U, N, and V.[2] We assume the following functional forms which are concave in their arguments as in Natalucci and Ravenna (2003) and Bokil (2005)

$$U(C_s^j) = \ln C_s^j \tag{2}$$

$$N(\frac{M_s^j}{P_s}) = \chi \ln \frac{M_s}{P_s} \tag{3}$$

$$V(L_s^j) = \vartheta \frac{(L_s^{S,j})^\kappa}{\kappa} \tag{4}$$

(2) implies that the agent is risk-averse and that the intertemporal elasticity of substitution and the intratemporal relative risk aversion coincide and are equal to unity. χ denotes the utility scale parameter for real money balances $\frac{M_s^j}{P_s}$ and ϑ is the respective parameter for total labour supply of household j, $L_s^{S,j}$. $\kappa > 1$ denots labour supply elasticity.

Labour supply of household j is mobile within the country and hence across the sectors and perfectly substitutable between H and N

$$L_s^{S,j} = L_{N,s}^j + L_{H,s}^j \tag{5}$$

$L_{N,s}^j$ denote hours worked in the N sector and $L_{H,s}^j$ denote hours worked in the H sector. We assume that the intratemporal elasticity of substitution between traded goods T and non-traded goods N is equal to one. We then obtain that preferences about total consumption are of the Cobb-Douglas form

$$C_s^j = \frac{(C_{T,s}^j)^\gamma (C_{N,s}^j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \tag{6}$$

Tradable consumption $C_{T,s}^j$ is divided between home produced goods H and foreign tradable goods F

$$C_{T,s}^j = \frac{(C_{H,s}^j)^\nu (C_{F,s}^j)^{1-v}}{\nu^\nu (1-\nu)^{1-\nu}} \tag{7}$$

where again the intratemporal elasticity of substitution between home produced H and foreign produced tradables F equals one. The latter ensures that the Law of One Price (LOP) will hold in the tradable sector. The non-tradable consumption basket is defined as the aggregate consumption over all non-tradables produced at home

$$C_{N,s}^j = [\int_0^1 (c_{N,s}^j(z))^{\frac{\rho-1}{\rho}} dz]^{\frac{\rho}{\rho-1}} \tag{8}$$

The consumption based price index (CPI) P_s results from minimising total expenditures of household j for obtaining 1 unit of the consumption index C_s^j over the arguments $C_{N,s}^j$ and $C_{T,s}^j$

$$P_s = (P_{T,s})^\gamma (P_{N,s})^{1-\gamma} \tag{9}$$

[2] Additive separability ensures that each marginal utility/disutility in one of the arguments does not depend on the respective other arguments.

By the same procedure we obtain the tradables price index

$$P_{T,s} = (P_{H,s})^v (P_{F,s})^{1-v} \tag{10}$$

and analogously

$$P_{N,s} = [\int_0^1 (p_{N,s}(z))^{1-\rho_N} dz]^{\frac{1}{1-\rho_N}} \tag{11}$$

3.1.2 Optimality conditions

The budget constraint for household j in nominal terms reads

$$\begin{aligned} B^j_{H,t-1} + S_t B^j_{F,t-1} + Q^j_t + M^j_{t-1} + \int_0^1 \Pi^j_{N,t}(z)dz + \int_0^1 \Pi^j_{H,t}(z)dz + W^j_{H,t}L^j_{H,t} + W^j_{N,t}L^j_{N,t} \\ +P_{N,t}R^{N,j}_t K^{N,j}_{t-1} + P_{H,t}R^{H,j}_t K^{H,j}_{t-1} \\ \geq P_t C^j_t + T_t + M^j_t + P^{I,H}_t I^{H,j}_t + P^{I,N}_t I^{N,j}_t + \frac{B^j_{H,t}}{1+i_t} + \frac{S_t B^j_{F,t}}{1+i^*_t} \end{aligned} \tag{12}$$

The household receives income from interest payments on home zero coupon bonds $B^j_{H,t-1}$ and foreign zero coupon bonds $S_t B^j_{F,t-1}$, lump-sum transfers from the government Q^j_t, from money holdings M^j_{t-1} and profits of firms. She receives factor income from supplying labour to firms in the H and N sector $W^j_{H,t}L^j_{H,t} + W^j_{N,t}L^j_{N,t}$ and renting out capital $P_{N,t}R^{N,j}_t K^{N,j}_{t-1} + P_{H,t}R^{H,j}_t K^{H,j}_{t-1}$. The revenues are used to finance total consumption and investment expenditures $P_t C^j_t + P^{I,H}_t I^{H,j}_t + P^{I,N}_t I^{N,j}_t$, pay lump-sum taxes T_t, carry money to the next period M^j_t, and purchase domestic and foreign bonds. As the private sector issues the bonds we can think of $B^j_{H,s}$ and $B^j_{F,s}$ as corporate bonds issued by the household-owned firms. Note that if $B^j_{H,s} < 0$, household j is a net debtor/issuer of the home bond. As the government does not have an objective function and the balance is part of the households balance, we can interpret the home corporate bond similarly as a bond issued by the government. As the intertemporal utility function U^j_t given by (1) has the usual property of local non-satiation as it is concave in its arguments and additively separable in per period utilities, effi ciency requires that the budget constraint (12) will hold with equality.

i^*_t denotes the nominal world interest rate at which domestic agents can buy/sell foreign assets. i^*_t is comprised of the world interest rate plus a risk premium increasing in net real foreign liabilities to be introduced below. i_t is the domestic nominal short rate. Home households possess the H and N firms and decide on investment spending and capital adjustment. Household j therefore has to take into account the law of accumulation of capital in the H and N sectors when deciding on the optimum capital stock K^J_t and period investment I^J_t, $J = H, N$. Capital accumulation therefore follows

$$K^N_t = \Phi(\frac{I^N_t}{K^N_{t-1}})K^N_{t-1} + (1-\delta)K^N_{t-1} \tag{13}$$

$$K^H_t = \Phi(\frac{I^H_t}{K^H_{t-1}})K^H_{t-1} + (1-\delta)K^H_{t-1} \tag{14}$$

Capital accumulation incurs concave adjustment costs $\Phi' > 0 > \Phi''$.

We end up with the following optimality conditions. Consumption between traded and non-traded goods is given by

$$\frac{C_{T,s}}{C_{N,s}} = \frac{\gamma}{1-\gamma}\frac{P_{N,s}}{P_{T,s}} \tag{15}$$

and analogously for the choice between home produced and foreign produced tradables basket

$$\frac{C_{H,s}}{C_{F,s}} = \frac{v}{1-v}\frac{S_t P^*_{F,s}}{P_{H,s}} = \frac{v}{1-v}\frac{S_t}{P_{H,s}} \tag{16}$$

where S_t denotes the nominal exchange rate (assumed to be fix). As the economy is small it cannot affect world prices and takes the foreign price level $P^*_{F,s}$ as given. We normalised the latter to 1.[3] The intertemporal consumption/savings decision is guided by the Euler equation

$$\lambda^C_t = \beta E_t[\lambda^C_{t+1}(1+i_t)\frac{P_t}{P_{t+1}}] \tag{17}$$

where $\lambda^C_t = (C^j_t)^{-1}$ is the marginal utility of total consumption. Hence

$$\frac{1}{C^j_t} = \beta E_t[\frac{1}{C^j_{t+1}}(1+i_t)\frac{P_t}{P_{t+1}}] \tag{18}$$

The assumed form for utility out of total consumption implies that utility from consumption is additively separable in H, N, F consumption. We then obtain that the Euler equations for the non-tradable and home-produced tradables consumption are

$$\frac{1}{C^j_{N,t}} = \beta E_t[\frac{1}{C^j_{N,t+1}}\frac{P^N_t}{P^N_{t+1}}(1+i_t)] \tag{19}$$

$$\frac{1}{C^j_{H,t}} = \beta E_t[\frac{1}{C^j_{H,t+1}}\frac{P^H_t}{P^H_{t+1}}(1+i_t)] \tag{20}$$

Further, the household decides on her total supply of labour according to

$$\vartheta(L^j_{H,t} + L^j_{N,t})^{\kappa-1}C^j_t = \frac{W_t}{P_t} \tag{21}$$

which says that the household provides labour up to the point where the marginal rate of substitution between consumption and leisure equals the real wage in terms of total consumption units. We have used the fact that due to mobility of labour between the sectors and the perfect substitutability of providing labour in the traded versus non-traded sector, nominal wages have to be the same in both sectors $W^H_t = W^N_t = W_t$. The trade-off between real money balances and total consumption is determined by

$$\chi(\frac{M^j_t}{P_t})^{-1} = \lambda^C_t - \beta E_t[\lambda^C_{t+1}\frac{P_t}{P_{t+1}}]$$

By using the Euler equation (17) we obtain the money demand equation

$$\chi\frac{(\frac{M^j_t}{P_t})^{-1}}{\lambda^C_t} = \frac{i_t}{1+i_t} \tag{22}$$

which states that the marginal rate of substitution between real money balances for consumption equals the discounted payoff on bond earnings which accrue in period $t+1$. Payoffs are discounted by using the stochastic discount factor derived from (18).

In order to obtain an effi cient investment and physical capital allocation, the household maximises utility with respect to the constraints (13) and (14) as well as the real household

[3] We could alternatively assume that the foreign price level follows a stable exogenous stochastic process.

balance. Investment in the non-traded sector should therefore be guided by

$$\begin{aligned}\lambda_t^C \frac{P_t^{I,N}}{P_t} Q_t^N &= \beta E_t[\lambda_{s+1}^C \frac{P_{N,s+1}}{P_{s+1}} R_{s+1}^N] \\ &\quad + \beta E_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,N}}{P_{t+1}} Q_{t+1}^N]\{[\Phi[\frac{I_{t+1}^N}{K_t^N}] - \Phi'[\frac{I_{t+1}^N}{K_t^N}]\frac{I_{t+1}^N}{K_t^N} + (1-\delta)\}\end{aligned} \tag{23}$$

where we have aggregated over all households. ${Q_t}^N$ denotes Tobins' Q in the non-traded sector defined as the market value of capital over its replacement cost. Analogously we obtain for the H sector

$$\begin{aligned}\lambda_t^C \frac{P_t^{I,H}}{P_t} Q_t^H &= \beta E_t[\lambda_{s+1}^C \frac{P_{H,s+1}}{P_{s+1}} R_{s+1}^H] \\ &\quad + \beta E_t[\lambda_{t+1}^C \frac{P_{t+1}^{I,H}}{P_{t+1}} Q_{t+1}^H]\{[\Phi[\frac{I_{t+1}^H}{K_t^H}] - \Phi'[\frac{I_{t+1}^H}{K_t^H}]\frac{I_{t+1}^H}{K_t^H} + (1-\delta)\}\end{aligned} \tag{24}$$

Capital compared to labour is immobile between the sectors within the country, i.e. ex-post rental rates of equipment in the H and N given by $R_{N,s}$ and $R_{H,s}$ can differ. Hence there is no ex-post rental price equalisation. In long-run equilibrium however, rental prices will equalise as is shown in section 4.

3.1.3 Private Sector Balance

The budget constraint for household j in nominal terms is given by (12). Aggregating over all home agents $j \in [0,1]$ at home we obtain the private sector balance:

$$\begin{aligned}& B_{H,t-1} + S_t B_{F,t-1} + Q_t + M_{t-1} + \Pi_{N,t} + \Pi_{H,t} + W_{H,t} L_{H,t} + W_{N,t} L_{N,t} \\ & + P_{H,t} R_t^H K_{t-1}^H + P_{N,t} R_t^N K_{t-1}^N \\ = \; & P_{T,t} C_{T,t} + P_{N,t} C_{N,t} + T_t + M_t + P_t^{I,H} I_t^H + P_t^{I,N} I_t^N + \frac{B_{H,t}}{1+i_t} + \frac{S_t B_{F,t}}{1+i_t^*}\end{aligned} \tag{25}$$

We have used the fact that all households make same optimum decisions as the representative/average household, as they face the same set of constraints and preferences (elasticities). Hence $C_{T,t}^j = C_{T,t}$ and $\int_0^1 C_{T,t}^j dj = \int_0^1 C_{T,t} dj = C_{T,t}$, and so on for the other variables. Note that only in the closed economy we would have that $\int_0^1 B_{H,s}^j dj = 0$, as the domestic asset market would have to clear in the domestic economy and there would be no possibility of holding home bonds abroad. Here we have that $B_{H,s} + B_{H,s}^* = 0$ or $B_{H,s} = -B_{H,s}^*$, where $B_{H,s}^*$ denote home bonds held at foreign.

3.2 Firms in the non-traded and traded goods sector

3.2.1 Domestic Production and Decision Problem

Both sectors are populated by a continuum of monopolistically competitive firms residing in the interval $[0,1]$. Imperfect substitutability of produced goods allows for price-setting power of firms over their product. Due to their negligible size they cannot influence the overall aggregate price level of the respective sector and take it as given. In the setup considered here we assume that domestic firms are wholly owned by home households and all profits are distributed to domestic residents in the form of dividends.[4]

[4] We will relax this assumption in a follow up paper for the traded-goods sector when we introduce foreign direct investment (FDI) which serves as an intermediate good in tradable goods production.

Each firm $z \in [0,1]$ in the tradable sector combines capital $K^H_{s-1}(z)$ and labour $L_{H,s}(z)$ according to the Cobb-Douglas production function

$$y_{H,s}(z) = A^H_s(z)(K^H_{s-1}(z))^{\alpha_H}(L_s(z))^{1-\alpha_H} \tag{26}$$

where $K^H_{s-1}(z) = \int_0^1 K^{H,j}_{s-1}(z)dj$ and $L_{H,s}(z) = \int_0^1 L^j_{H,s}(z)dj$. $A^H_s(z) = A^H_s$ is total factor productivity in the tradable sector to be the same for all firms and follows an exogenous AR(1) stochastic process. Cost minimisation yields the standard factor demands

$$\frac{W_s}{P_{H,s}} = MC^H_s(1-\alpha_H)\frac{y_{H,s}(z)}{L_{H,s}(z)} \tag{27}$$

$$R^H_s = MC^H_s\alpha_H\frac{y_{H,s}(z)}{K_{H,s-1}(z)} \tag{28}$$

and analogously in the non-traded sector

$$\frac{W_s}{P_{N,s}} = MC^N_s(1-\alpha_N)\frac{y_{N,s}(z)}{L^N_s(z)} \tag{29}$$

$$R^N_s = MC^N_s\alpha_N\frac{y_{N,s}(z)}{K_{H,s-1}(z)} \tag{30}$$

where MC^J_s denotes real marginal cost in the respective sector $J = H, N$. Note that in both sectors, firms base their labour demand on the real wage obtained by deflating the nominal wage by the sector-specific aggregate price levels $P_{H,s}$ and $P_{N,s}$, respectively.

Labour supply is steered by the consumption based real wage which can be seen from (21). Finally, the labour market clears at all dates

$$L^S_s = \int_0^1 \{L^j_{H,s} + L^j_{N,s}\}dj = L^D_s = L_s$$

where for the sector-specific labour supplies

$$L^j_{H,s} = \int_0^1 L^j_{H,s}(z)dz$$

$$L^j_{N,s} = \int_0^1 L^j_{N,s}(z)dz$$

3.2.2 Calvo-Pricing in the H and N sector

We go through the reasoning for the traded sector only. Results for the non-traded goods sector are derived analogously. As a monopolistic competitor, each firm in the traded goods sector has to take into account the demand for its good. It can only sell more by lowering its price. Demand for good z consists of consumption and investment demand of any household j given by $c^j_{H,s}(z)$, $i^j_{H,s}(z)$, $j \in [0,1]$, government demand $g_{H,s}(z)$, and foreign demand $c^*_{H,s}(z)$ (exports)

$$c^j_{H,s}(z) = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H}C^j_{H,s}$$

$$i^j_{H,s}(z) = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H}I^j_{H,s}$$

$$c^*_{H,s}(z) = (\frac{p^*_{H,s}(z)}{P^*_{H,s}})^{-\rho^*_H} C^*_{H,s} = (\frac{p_{H,s}(z)/s_s(z)}{P_{H,s}/S_s})^{-\rho_H} C^*_{H,s}$$
$$g_{H,s}(z) = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H} G_s$$

We have used that the law of one price holds for each firm in the tradable sector $p^*_{H,s}(z) = p_{H,s}(z)/s_s(z)$ which requires that the price elasticities of domestic and foreign demand concerning tradables are equal, $\rho^*_H = \rho_H$. Further $s_s(z) = S_s$ i.e. the nominal exchange rate is to be given for any home producer and therefore in aggregate $P^*_{H,s} = P_{H,s}/S_s$ and eventually $c^*_{H,s}(z) = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H} C^*_{H,s}$. Aggregating over all households and noting that they are alike concerning preferences and constraints we obtain the overall domestic private sector consumption and investment demand for good z

$$c_{H,s}(z) = \int_0^1 c^j_{H,s}(z) dj = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H} C_{H,s}$$
$$i_{H,s}(z) = \int_0^1 i^j_{H,s}(z) dj = (\frac{p_{H,s}(z)}{P_{H,s}})^{-\rho_H} I_{H,s}$$

Firm z sets the price such that output $y_{H,t}(z)$ meets overall demand

$$y_{H,t}(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H} (C_{H,t} + C^*_{H,t} + G_{H,t} + I_{H,t}) \tag{31}$$
$$y_{H,t}(z) = (\frac{p_{H,t}(z)}{P_{H,t}})^{-\rho_H} Y_{H,t} \tag{32}$$

where we have set the date $s = t$.

The objective of the firm in each sector is to maximise the expected discounted flow of future profits. We assume Calvo (1983) pricing in the sense that each firm faces an exogenous and fixed probability in the traded sector $(1 - \theta_H)$ and $(1 - \theta_N)$ in the non-traded sector of being able to reset prices in any period. When re-setting the price of the product it has to take into account that the price has to remain optimal, given that there will be no resetting possibility till period $t + s$. The decision problem of the traded sector firm can then be stated as follows

$$\max_{\{p_{H,t}(z)\}} E_t \sum_{s=0}^{\infty} (\theta_H)^s \Delta_{s,t+s} \{\frac{p_{H,t}(z)}{P_{t+s}} y_{H,t+s}(z) - MC^H_{t+s} y_{H,t+s}(z)\}$$
$$\text{s.t. } y_{H,t+s}(z) = (\frac{p_{H,t}(z)}{P_{H,t+s}})^{-\rho_H} Y_{H,t+s}$$
$$\text{s.t. } P_{H,t}{}^{(1-\rho_H)} = (1 - \theta_H)(p^o_H)^{(1-\rho_H)} + \theta_H P^{(1-\rho_H)}_{H,t-1}$$

The later condition states that at each point in time the home-traded price level is a weighted average of prices chosen by firms that re-set prices in t and those that could not. $\Delta_{s,t+s} = E_t[\beta^s (\frac{C_{t+s}}{C_t})^{-1}]$ denotes the stochastic discount factor used for evaluating the expected future profit streams by the firm at date t. Optimal price setting by firm z then becomes

$$\frac{p^o_{H,t}(z)}{P_{H,t}} = \frac{\rho_H}{\rho_H - 1} \frac{E_t \sum_{s=0}^{\infty} (\theta_H \beta)^s (C_{t+s})^{-1} MC^H_{t+s} (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H} Y_{H,t+s}}{E_t \sum_{s=0}^{\infty} (\theta_H \beta)^s (C_{t+s})^{-1} (\frac{P_{H,t+s}}{P_{H,t}})^{\rho_H - 1} Y_{H,t+s}} \tag{33}$$

3.3 Hybrid Inflation dynamics

Inertia in the price level dynamics is introduced by assuming that there is a share of ω_H firms in the H sector and a share of ω_N firms in the N sector as well as of ω_F firms in the foreign country which are backward looking as in Gali and Gertler (1999). They are backward-looking in the sense that they have to use last periods' optimally set prices and the change in the aggregate price level as informational sources when resetting prices. For the H sector the price set by a backward looking firm is therefore

$$P^b_{H,t} = P^{\#}_{H,t-1} \frac{P_{H,t-1}}{P_{H,t-2}} \tag{34}$$

The aggregate price level in the tradable sector then evolves according to

$$P_{H,t} = \left[(1-\theta_H) P^{\#,1-\rho_H}_{H,t} + \theta_H P^{1-\rho_H}_{H,t-1} \right]^{\frac{1}{1-\rho_H}} \tag{35}$$

$(1-\theta_H)$ is the exogenous probability that a firm can re-set prices in t which is then split between firms that optimise and those that do not when selecting a new output price. Analogous relationships hold in the N sector and for tradables production at foreign. $P^{\#}_{H,t}$ is an index of newly (optimally) set prices by forward- and backward looking firms

$$P^{\#}_{H,t} = \left[(1-\omega_H) P^{f,1-\rho_H}_{H,t} + \omega_H P^{b,1-\rho_H}_{H,t} \right]^{\frac{1}{1-\rho_H}} \tag{36}$$

$P^{\#}_{H,t}$ contains newly set prices of all firms that re-set prices in t, be they forward- (optimising) or backward- (non-optimising) looking. Log-linearising the above three indices around P_t yields

$$\begin{aligned} p^b_{H,t} &= p^{\#}_{H,t-1} + \pi_{H,t-1} - \pi_{H,t} \\ p_{H,t} &= (1-\theta_H)\, p^{\#}_{H,t} + \theta_H p_{H,t-1} \\ p^{\#}_{H,t} &= (1-\omega_H)\, p^f_{H,t} + \omega_H p^b_{H,t} \end{aligned}$$

Analogous relationships hold in the N sector. After log-linearisation we end up with the following sector-specific hybrid inflation dynamics

$$\pi_{H,t} = \lambda^b_H \pi_{H,t-1} + \lambda^{mc}_H mc_{H,t} + \lambda^f_H E_t \pi_{H,t+1} \tag{37}$$

$$\pi_{N,t} = \lambda^b_N \pi_{N,t-1} + \lambda^{mc}_N mc_{N,t} + \lambda^f_N E_t \pi_{N,t+1} \tag{38}$$

The deep parameters are $\lambda^b_{H,t} = \frac{\omega_H}{\theta_H + \omega_H(1-\theta_H(1-\beta))}$, $\lambda^{mc_H}_{H,t} = \frac{(1-\omega_H)(1-\theta_H)(1-\theta_H\beta)}{\omega_H(1-\theta_H+\theta_H\beta)+\theta_H}$, $\lambda^f_{H,t} = \frac{\beta\theta_H}{\omega_H}\lambda^b_{H,t}$ and analogously for the N sector. For the share of backward looking firms $\omega_H, \omega_N \to 0$ we obtain the forward-looking New Keynesian Phillips curves.[5]

3.4 Central Bank

The central bank receives income from seigniorage earnings $M_t - M_{t-1}$ when issuing base money M_t and interest earnings on one period foreign discount bonds $S_t B^C_{F,t-1}$. The monetary authority is not allowed to purchase/sell bonds denominated in home currency, in other words domestic credit has to be fully covered by the stock of foreign reserves. Therefore, the monetary base M_t as well as the short run domestic interest rate i_t are endogenous and cannot be influenced. Revenues are used to purchase $B^C_{F,t}$ units of foreign pure discount bond which cost $\frac{S_t B^C_{F,t}}{1+i^*_t}$ in home currency and mature at beginning of period $t+1$. The central bank balance therefore

[5] For a detailed derivation of the hybrid inflation dynamics please refer e.g. to Holmsberg (2006).

reads

$$M_t - M_{t-1} = Z_t - Z_{t-1} + \frac{S_t B^C_{F,t}}{1+i^*_t} - S_t B^C_{F,t-1} + v_t \tag{39}$$

$B^C_{F,t}$ denotes the quantity of nominal pure discount bonds held by the central bank that are issued at end of t and pay out 1 unit of foreign currency at the beginning of $t+1$. Changes in foreign reserves holdings $Z_t - Z_{t-1}$ arising from movements in the balance of payments are included on the asset side (right hand side) of the balance. $\frac{S_t B^C_{F,t}}{1+i^*_t} - S_t B^C_{F,t-1}$ are net nominal foreign assets of the central bank. $v_t > 0$ are nominal net transfers of the central bank to the government after bond purchases and money supply changes are accomplished. As v_t has no sign restriction, transfers can go either way and so this variable resembles the possibility that the government can deposit revenues in the central bank (which are liabilities to the central bank and have to be fully covered by foreign reserve holdings). Hence there is theoretically a way of financing of government deficits by monetisation. As our model does not feature public debt and the fiscal stance is to be balanced every period this channel of discretionary monetary policy is closed. We see that one way of financing foreign bond purchases is by period t lump sum taxation of the government.

Under the automatic currency board domestic base money M_t at any point in time is fully backed by foreign reserve holdings

$$Z_t \geq M_t$$

where effi ciency requires that

$$Z_t = M_t \tag{40}$$

which yields the result that at any point in time the stock of currencies and coins in circulation evolves according to

$$M_t = M_{t-1} + Z_t - Z_{t-1} \tag{41}$$

(40) is the "policy rule" of the central bank under the currency board and closes the model.[6] The domestic base money supply adjusts mechanically to balance of payments deficits/surpluses and the central bank cannot sterilise this impact as it would be possible under a fixed exchange rate system (by changing home reserves in the opposite direction).We further see from (39) that the rule implies that $S_t B^C_{F,t-1} = \frac{S_t B^C_{F,t}}{1+i^*_t} + v_t$. Solving the latter forward we obtain

$$B^C_{F,t-1} = (\prod_{s=t}^{T} \frac{1}{1+i^*_s}) B^C_{F,s} + \sum_{s=t}^{T} (\prod_{j=t}^{s-1} \frac{1}{1+i^*_j}) \frac{v_s}{S_s}$$

with $\prod_{j=t}^{t-1} \frac{1}{1+i^*_j} \equiv 1$. Ruling out Ponzi-schemes, i.e. imposing the condition that the initial central bank assets have to be redeemed sooner or later requires

$$\lim_{T\to\infty} (\prod_{s=t}^{T} \tfrac{1}{1+i^*_s}) B^C_{F,T} = 0$$

The initial asset position is therefore (ex post[7]) sustainable if

$$B^C_{F,t-1} = \sum_{s=t}^{\infty} (\prod_{j=t}^{s-1} \frac{1}{1+i^*_j}) \frac{v_s}{S_s}$$

[6] Under an independent central bank, (40) would be replaced by an interest rate setting rule/inflation targeting rule.

[7] I.e. after all values have realised and expectation operators can be omitted.

Hence an initial net asset position in deficit has to be matched by the present discounted value of all future government transfers to be positive ($v_t < 0$) and vice versa.

3.5 Government

In our benchmark model, government consumption "goes into the ocean" in the sense that it does not provide any utility to households. Further, revenues are not used to cure ineffi ciencies in the economy, i.e. there is no offsetting of ineffi ciencies in equilibrium output caused by pricing power of monopolistically competitive firms. Under this assumption the government balance is given by

$$T_t + v_t = P_{H,t}G_{H,t} + P_{N,t}G_{N,t} + Q_t \tag{42}$$

Expenditures cover government purchases on home produced tradables $P_{H,t}G_{H,t}$ and non-tradables $P_{N,t}G_{N,t}$ as well as lump sum transfers $Q_t = \int_0^1 Q_t^j dj$ to households (social benefits). Further we have assumed that government expenditures on the tradable good only fall on home produce. Lump sum taxation $T_t = \int_0^1 T_t^j dj$ and transfers from the central bank captured by v_t serve as source of financing.

Again we restrain from the acknowledged fact, that the design of the Bulgarian currency board entails, whether intentionally or not, the possibility that the government conducts monetary policy through its deposits in the liability side of the issue department covered by v_t in our model. So far, there is no policy rule in the model other than to immediately spend what is earned from lump sum taxation and interest income on foreign-currency denominated bonds.[8]

3.6 Foreign Sector

3.6.1 Uncovered Interest Parity

Maximising expected utility (1) with respect to $\{B_{H,t}, B_{F,t}\}$ in the private sector balance (25) yields the uncovered interest parity condition

$$E_t[\lambda_{t+1}^C \frac{P_t}{P_{t+1}}\{(1+i_t) - (1+i_t^*)\frac{S_{t+1}}{S_t}\}] = 0$$

Using that from the Euler equation (18) we get $E_t[\lambda_{t+1}^C \frac{P_t}{P_{t+1}}] = \frac{\lambda_t^C}{\beta(1+i_t)}$ and simplifying

$$\begin{aligned} 0 &= E_t[\frac{\lambda_t^C}{\beta(1+i_t)}\{(1+i_t) - (1+i_t^*)\frac{S_{t+1}}{S_t}\}] \\ (1+i_t) &= (1+i_t^*)E_t\frac{S_{t+1}}{S_t} \end{aligned} \tag{43}$$

In our model UIP holds up to a risk premium. The interest rate at which home household can borrow/lend internationally (for which the UIP holds) and the exogenous world interest rate are linked by

$$(1+i_t^*) = (1+\tilde{i}_t^*)\phi[-\frac{F_t}{P_t}] \tag{44}$$

where $\frac{F_t}{P_t}$ denote net real foreign assets ($\frac{F_t}{P_t} >= 0$) and net real foreign liabilities ($\frac{F_t}{P_t} < 0$), respectively. Equation (44) ensures the stationarity of the economy when exposed to temporary shocks, e.g. a world interest rate shock.[9] We further assume that in the (initial) equilibrium the interest rates converge which requires that $\phi[-\frac{F}{P}] = 1$. Therefore the risk premium is 0 in steady state and all short run rates in the model equalise.

[8]Note that seignorage earnings cannot be spent as under the currency board they mechanically feed into changes in foreig reserves holdings, as (41) makes clear.

[9]See also Schmitt-Grohe and Uribe (2003) for this point.

3.6.2 Terms of Trade and the Internal Real Exchange Rate

By definition the consumption based internal real exchange rate in levels is given by the domestic currency price of one unit of the tradables basket in units of non-tradables

$$Q_t = \frac{P_{T,t}}{P_{N,t}} \tag{45}$$

The terms of trade are given by the relative price of imported goods in terms of home produced tradables

$$T_t = \frac{P_{F,t}}{P_{H,t}} = \frac{S_t}{P_{H,t}} \tag{46}$$

Q_t and T_t can be used to show how changes in the external competitiveness affect domestic real marginal cost in each sector. From (27) and (29) we directly obtain

$$\frac{P_{N,t}}{P_{H,t}} = \frac{(1-\alpha_H)}{(1-\alpha_N)} \frac{MC_s^H}{MC_s^N} \frac{Y_{H,t}}{Y_{N,t}} \frac{L_s^N}{L_s^H} \tag{47}$$

or

$$\frac{T_t^{1-v}}{Q_t} = \frac{(1-\alpha_H)}{(1-\alpha_N)} \frac{MC_s^H}{MC_s^N} \frac{Y_{H,t}}{L_s^H} \frac{L_s^N}{Y_{N,t}} \tag{48}$$

A temporary/permanent real depreciation (an increase in Q_t) - ceteris paribus - decreases real marginal cost in the tradable sector. The same effect can be attained by an improvement (a decrease) in the terms of trade T_t. Furthermore, an increase in the productivity of labour in the traded sector causes a real appreciation in Q_t / a deterioration (an increase) in the terms of trade.

Our model entails the Balassa-Samuelson effect which can be seen from log-linearising (47) around a zero inflation steady state

$$y_{H,t} - l_t^H + mc_t^H + p_{H,t} = y_{N,t} - l_t^N + mc_t^N + p_{N,t}$$

An increase in labour productivity in the traded-goods sector $y_{H,t} - l_t^H$ will lead to an increase in real marginal cost in the non-traded sector leading to higher inflation in the non-traded goods sector. The latter can be seen from the hybrid Phillips-curve given by (38). Furthermore, we can write the CPI price level (9) as

$$P_t = S_t(Q_t)^{\gamma-1}(T_t)^{-v} \tag{49}$$

from which it follows that cet. par. a depreciation of the real exchange rate (a rise in Q_t for the share of tradables $\gamma < 0.5$) or a deterioration of the terms of trade decreases the price level and vice versa.

3.6.3 Current Account

In an open economy framework, the difference between total income and domestic consumption is defined as the current account. To obtain total income, we write the resource constraints of the home economy in nominal terms

$$\begin{aligned} P_{H,t}Y_{H,t} &= P_{H,t}C_{H,t} + P_{H,t}I_{H,t} + P_{H,t}G_{H,t} + P_{H,t}C^*_{H,t} \\ P_{N,t}Y_{N,t} &= P_{N,t}C_{N,t} + P_{N,t}I_{N,t} + P_{N,t}G_{N,t} \end{aligned}$$

In our framework, investment and final output in a sector face the same price elasticities of demand and hence have the same equilibrium price $P_t^{I,N} I_t^N = P_t^N I_t^N$ and $P_t^{I,H} I_t^H = P_t^H I_t^H$.[10] Further note that $P_{T,t}C_{T,t} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. To obtain the current account, we add up (25), (42), (39) and take into account that aggregate equilibrium profits in both sectors are

$$\begin{aligned} \Pi_{H,t} &= P_{H,t}Y_{H,t} - W_{H,t}L_{H,t} - P_{H,t}R_t^H K_{t-1}^H \\ \Pi_{N,t} &= P_{N,t}Y_{N,t} - W_{N,t}L_{N,t} - P_{N,t}R_t^N K_{t-1}^N \end{aligned}$$

Hence

$$\begin{aligned} & B_{H,t-1}+S_tB_{F,t-1}+M_{t-1} + T_t + v_t+Q_t + S_tB_{F,t-1}^C + M_t - M_{t-1} \\ & +P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} \\ = \; & P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{H,t}I_t^H + P_{H,t}G_{H,t} + P_{N,t}C_{N,t} + P_{N,t}I_t^N + P_{N,t}G_{N,t} + T_t \\ & +v_t + Q_t + M_t + \frac{B_{H,t}}{1+i_t} + \frac{S_tB_{F,t}}{1+i_t^*} + \frac{S_tB_{F,t}^C}{1+i_t^*} + Z_t - Z_{t-1} \end{aligned}$$

We eventually obtain

$$\begin{aligned} & B_{H,t-1}+S_tB_{F,t-1}+S_tB_{F,t-1}^C+P_{H,t}C_{H,t}^* \\ = \; & P_{F,t}C_{F,t}+\frac{B_{H,t}}{1+i_t}+\frac{S_tB_{F,t}}{1+i_t^*}+\frac{S_tB_{F,t}^C}{1+i_t^*}+Z_t-Z_{t-1} \end{aligned}$$

The nominal current account is then the left hand side of the following equation

$$B_{H,t-1}+S_tB_{F,t-1}+S_tB_{F,t-1}^C+NX_t=\frac{B_{H,t}}{1+i_t}+\frac{S_t(B_{F,t}+B_{F,t}^C)}{1+i_t^*}+Z_t-Z_{t-1} \tag{50}$$

where $NX_t = P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t}$ denotes the nominal trade balance expressed in home currency. On the left hand side we have that the current account is comprised of factor income on bonds bought at the end of period $t-1$ and paid out at the beginning of t as well as net trade.

3.6.4 Balance of payments

The Balance of Payments (BOP) is defined as

$$BOP_t \equiv CA_t + FA_t - ZZ_t = 0$$

After rearranging (50) we obtain the nominal balance of payments:

$$\begin{aligned} & BOP_t \\ \equiv \; & B_{H,t-1}+S_tB_{F,t-1}+S_tB_{F,t-1}^C+NX_t+\left(-\frac{B_{H,t}}{1+i_t}-\frac{S_t(B_{F,t}+B_{F,t}^C)}{1+i_t^*}\right) - (Z_t - Z_{t-1}) = 0 \end{aligned} \tag{51}$$

where CA_t denotes the current account, FA_t denotes the financial account and ZZ_t denotes the foreign reserves/exchange balance. $ZZ_t - FA_t$ equals the change in the net asset position of the economy which is equal to the current account. A (temporary) BOP surplus $BOP_t > 0$ induced by a current account surplus (an increase in net exports/net factor income) or a financial account deficit (a net capital export) implies that there is a +ve change in net assets. Then there must be a net increase in the foreign reserves balance, $ZZ_t > 0$ which under the currency board translates into an increase in domestic base money supply. Analogously, net capital imports

[10] A follow up paper relaxes this assumption, in order to study FDI as intermediate good of production, see also Natalucci and Ravenna (2003).

(+ve FDI inflows) cause foreign reserve inflows and have a expansionary impact on domestic money supply. We see this mechanism directly by employing the policy rule (40)

$$BOP_t \equiv CA_t + FA_t = M_t - M_{t-1}$$

Denote net nominal foreign assets by $F_t = \frac{B_{H,t}}{1+i_t} + \frac{S_t(B_{F,t}+B^C_{F,t})}{1+i^*_t}$ ie assets acquired at the end of period t which mature at beginning of period $t+1$. In our case net nominal foreign assets equal the -ve financial account $F_t = -FA_t$, as the full principal of last period's bond purchases is paid out as interest income and therefore only enters the current, but not the financial account. Use the UIP given by (43) to obtain

$$\begin{aligned}
F_t &= \frac{B_{H,t}S_t}{(1+i^*_t)E_tS_{t+1}} + \frac{S_t(B_{F,t}+B^C_{F,t})}{(1+i^*_t)}\frac{E_tS_{t+1}}{E_tS_{t+1}} \\
F_{t-1} &= \frac{B_{H,t-1}S_{t-1}}{(1+i^*_{t-1})E_{t-1}S_t} + \frac{S_{t-1}(B_{F,t-1}+B^C_{F,t-1})E_{t-1}S_t}{(1+i^*_{t-1})E_{t-1}S_t} \\
F_{t-1}(1+i^*_{t-1})\frac{E_{t-1}S_t}{S_{t-1}} &= B_{H,t-1} + E_{t-1}S_tB_{F,t-1} + E_{t-1}S_tB^C_{F,t-1}
\end{aligned}$$

As returns on period $t-1$ bond holdings are paid out at beginning of period t when S_t has realised, we can drop expectations[11]

$$F_{t-1}(1+i^*_{t-1})\frac{S_t}{S_{t-1}} = B_{H,t-1} + S_tB_{F,t-1} + S_tB^C_{F,t-1}$$

Rewrite the balance of payments (51) to obtain the evolvement of nominal net foreign assets

$$F_{t-1}(1+i^*_{t-1})\frac{S_t}{S_{t-1}} + NX_t - F_t = Z_t - Z_{t-1}$$

$$\begin{aligned}
F_t &= (1+i^*_{t-1})\frac{S_t}{S_{t-1}}F_{t-1} + NX_t - (Z_t - Z_{t-1}) \\
F_t &= (1+i_{t-1})F_{t-1} + NX_t - (Z_t - Z_{t-1}) \qquad (52)
\end{aligned}$$

where we have that if $F_t > 0$, home country has a positive net foreign asset position to abroad and vice versa. We can rewrite the nominal financial foreign asset position given by (52) to obtain a condition for the intertemporal solvency of the home country. We deflate by the CPI price index to obtain the real BOP in period t

$$\begin{aligned}
\frac{F_t}{P_t} &= (1+i_{t-1})\frac{F_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}} + \frac{NX_t}{P_t} - \left(\frac{Z_t}{P_t} - \frac{Z_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}}\right) \\
F^r_t &= \frac{1+i_{t-1}}{1+\pi_t}F^r_{t-1} + NX^r_t - \left(Z^r_t - \frac{Z^r_{t-1}}{1+\pi_t}\right) \qquad (53)
\end{aligned}$$

Solving forward yields the condition

$$F^r_{t-1} = \sum_{s=t}^{\infty}\left(\prod_{j=t}^{s}\left(\frac{1+i_{j-1}}{1+\pi_j}\right)^{-1}\right)\left\{Z^r_s - \frac{Z^r_{s-1}}{1+\pi_s} - NX^r_s\right\} \qquad (54)$$

where we have ruled out Ponzi-schemes. The intertemporal net asset position gives guidance for the intertemporal sustainability of current account deficits under the currency board. If

[11] We have not fixed the nominal exchange rate S_t in this section to allow for comparing our results with the results under a small open economy under a flexible rate.

the economy is initially (in the initial steady state at date $t-1$) a net borrower from abroad, i.e. $F^r_{t-1} < 0$ it needs to sell real foreign reserves and/or attain net real trade surpluses. This condition accounts for the fact that the economy - as typical under the economic transition process - maintains a current account deficit for some period as it does not need to be fulfilled every period. The condition only states that the presented discounted income flow from selling reserves/increases in net-trade have in sum to be positive and large enough to pay off initial net liabilities to abroad.

4 The Steady State

The steady state describes the long-run deterministic equilibrium of the economy where all prices are flexible and optimum decisions of households and firms are guided by relative prices only. In order to show that the long-run equilibrium is stationary and unique, we explain all the (ratios of) endogenous and state variables by exogenous parameters only.

4.1 Pricing Decision of Firms and Steady State Price Levels

In steady state, all prices are flexible and therefore the price-resetting probability for forward-looking firms is $1-\theta_H = 1-\theta_N = 1$. We obtain steady state values by omitting the time subscript from the variables. From (34) - (36) directly follows that

$$\begin{aligned} P^b_H &= P^{\#}_H \\ P_H &= P^{\#}_H \end{aligned}$$

Plugging in (36)

$$\begin{aligned} \left(P^b_H\right)^{1-\rho_H}(1-\omega_H) &= (1-\omega_H)\left(P^f_H\right)^{1-\rho_H} \\ P^b_H &= P^f_H \end{aligned}$$

which coincides with the result if there would be no backward-looking firms at all, ie $\omega_H = 0$. The analogous result is obtained for the N sector.

With monopolistically competitive firms, the steady state does however not yield the first best outcome. Market power causes equilibrium prices to be above those of the competitive outcome where all firms are price takers which can be seen from

$$\frac{p^o_H(z)}{P_H} = 1 = \frac{\rho_H}{\rho_H - 1} MC^H > MC^H \tag{55}$$

$$\frac{p^o_N(z)}{P_N} = 1 = \frac{\rho_N}{\rho_N - 1} MC^N > MC^N \tag{56}$$

where $\frac{\rho_H}{\rho_H-1} > 1$ and $\frac{\rho_N}{\rho_N-1} > 1$ are the sector-specific Lerner-indices of monopoly power and indicate the mark-up charged over real marginal cost. As the price elasticities of demand $1 < \rho_H$, $\rho_N < \infty$, firms receive a mark-up over real marginal costs. For $\rho_H, \rho_N \to \infty$ we would obtain the competitive "first-best" result in both sectors where output prices reflect real marginal cost.

We further assume - without loss of generality - that $P_H = P_N = S = W = 1$.[12] We earlier assumed that $P^*_{F,t} = 1$ and by the LOP $P^*_{F,t} = \frac{P_{F,t}}{S_t}$. Hence $P^*_F = P_F = 1$ and $p^o_H(z) = p^o_N(z) = 1$ for all firms z. Steady state price indices are then given by

$$\begin{aligned} P_T &= (P_H)^{v}(P_F)^{1-v} = 1 \\ P &= (P_T)^{\gamma}(P_N)^{1-\gamma} = 1 \end{aligned}$$

[12] Note that fixing prices at any other arbitrary level would be in line with a zero inflation rate steady state.

4.2 The Real Sector

We obtain from the Euler equation given by (17) that subjective and market discounting are linked by

$$i = \frac{1-\beta}{\beta} \tag{57}$$

We further find that capital accumulation in steady state is

$$\begin{aligned}
K^N &= \Phi(\frac{I^N}{K^N})K^N + (1-\delta)K^N \\
K^H &= \Phi(\frac{I^H}{K^H})K^H + (1-\delta)K^H \\
\delta &= \Phi(\frac{I^N}{K^N}) \\
\delta &= \Phi(\frac{I^H}{K^H})
\end{aligned}$$

For the functional form of Φ we assume as in Gali et al. (2004) that $\Phi(\delta) = \delta$ from which follows that $\Phi'(\delta) = 1$ and that $\delta = \frac{I^H}{K^H} = \frac{I^N}{K^N}$. From the latter we see that in long-run equilibrium the share of new investment to physical capital just equals the depreciation rate of existing capital in order to leave the capital stock of the economy constant. Nominal and real rate of returns are linked by

$$(1+i) = \frac{\beta}{1-\beta(1-\delta)}R^H = \frac{\beta}{1-\beta(1-\delta)}R^N$$

As both sectors face the same depreciation rate δ of real capital, real returns on physical capital have to equalise in steady state. From the resource constraint in the tradable sector

$$1 = \frac{C_H}{Y_H} + \delta\frac{K_H}{Y_H} + \frac{G_H}{Y_H} + \frac{C_H^*}{Y_H}$$

By the factor demands for physical capital (28) and (30) we obtain the capital output ratios

$$\frac{K_H}{Y^H} = (\frac{1}{\alpha_H}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_H}{\rho_H - 1})^{-1} \tag{58}$$

$$\frac{K_N}{Y^N} = (\frac{1}{\alpha_N}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_N}{\rho_N - 1})^{-1} \tag{59}$$

where we used that firms' real marginal costs in steady state is $\frac{\rho_H - 1}{\rho_H}$ and $\frac{\rho_N - 1}{\rho_N}$ respectively. Hence steady state investment/capital ratios are the same in both sectors. From the labour demands given by (27) and (29) we recover the labour shares

$$\frac{L_H}{Y_H} = (\frac{1}{1-\alpha_H}\frac{\rho_H}{\rho_H - 1})^{-1} \tag{60}$$

$$\frac{L_N}{Y^N} = (\frac{1}{1-\alpha_N}\frac{\rho_N}{\rho_N - 1})^{-1} \tag{61}$$

From aggregate supply in the economy we obtain the capital-labour ratio in both sectors

$$\begin{aligned}
\frac{K^H}{L^H} &= (\frac{Y_H}{L^H})^{\frac{1}{\alpha_H}} = (\frac{1}{1-\alpha_H}\frac{\rho_H}{\rho_H - 1})^{\frac{1}{\alpha_H}} \\
\frac{K^N}{L^N} &= (\frac{Y_N}{L^N})^{\frac{1}{\alpha_H}} = (\frac{1}{1-\alpha_N}\frac{\rho_N}{\rho_N - 1})^{\frac{1}{\alpha_N}}
\end{aligned}$$

where we have aggregated over the average firms̓ production and used that by assumption $A^H = A^N = 1$. The steady state consumption shares $\frac{C_H}{Y_H}$ and $\frac{C_N}{Y_N}$ are uniquely determined by exogenous variables and parameters which follows from

$$\begin{aligned}
1 &= \frac{C_H}{Y_H} + \delta(\frac{1}{\alpha_H}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_H}{\rho_H-1})^{-1} + \frac{G_H}{Y_H} + \frac{C_H^*}{Y_H} \\
\frac{C_H}{Y_H} &= 1 - \delta(\frac{1}{\alpha_H}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_H}{\rho_H-1})^{-1} - \frac{G_H}{Y_H} - \frac{C_H^*}{Y_H} \qquad (62) \\
\frac{C_N}{Y_N} &= 1 - \delta(\frac{1}{\alpha_N}\frac{1-\beta(1-\delta)}{\beta}\frac{\rho_N}{\rho_N-1})^{-1} - \frac{G_N}{Y_N} \qquad (63)
\end{aligned}$$

where the government shares $\frac{G_H}{Y_H}$, $\frac{G_N}{Y_N}$ and the export quota $\frac{C_H^*}{Y_H}$ are set exogenously.

4.3 Current Account and Balance of Payments

In (51) we obtained the balance of payments and that in steady state

$$\begin{aligned}
B_H + SB_F + SB_F^C + NX + (-\frac{B_H}{1+i} - \frac{S(B_F + B_F^C)}{1+i^*}) - (Z - Z) &\equiv 0 \qquad (64) \\
\frac{i}{1+i}B_H + \frac{i^*}{1+i^*}(B_F + B_F^C) &= -NX \qquad (65)
\end{aligned}$$

From the risk premium equation (44) follows that $\phi[F] = 1$. Hence in steady state home households can borrow at the world interest rate

$$i^* = \tilde{\imath}^*$$

and all nominal interest rates equalise, $i = i^* = \tilde{\imath}^*$. Then steady state net financial assets can be written as

$$\frac{\tilde{\imath}^*}{1+\tilde{\imath}^*}(B_H + B_F + B_F^C) = -NX$$

We can express this steady state relationship in terms of units of the home produced tradable output

$$\frac{\tilde{\imath}^*}{1+\tilde{\imath}^*}(\frac{B_H + B_F + B_F^C}{Y_H}) = -(\frac{C_H^* - C_F}{Y_H})$$

Real net financial assets equal nominal net financial assets in steady state and can be obtained by omitting all time subscripts from (53)

$$\frac{NX}{F} = \frac{\beta - 1}{\beta} < 0 \qquad (66)$$

The latter equation implies that if the steady state financial assets F are -ve they have to be offset by a +ve net trade balance NX in order to yield a balanced net financial asset account in long-run equilibrium and vice versa.[13] Then the steady state reserves to asset ratio $\frac{Z}{F}$ has to be -ve as well. In other words in steady state the current account has to be balanced. Otherwise we would have indeterminacy of the long-run equilibrium.

[13] We could have also imposed long-run equilibrium on (54) to obtain the same steady state relationship.

5 Calibration

We calibrate our model for quarterly data.[14] Regarding preferences, households̉ discount factor is set to $\beta = 0.99$ implying a quarterly zero inflation steady state interest rate of $\frac{1}{\beta}100\% = 1.01\%$ and an effective annual interest rate of 4.1%. We set the Frisch elasticity of labour supply $\frac{1}{\kappa}$ to $\frac{1}{2}$ as in Natalucci and Ravenna (2003). Following Valev (2005) who relies on data from the 1997 Bulgarian input-output matrix we assume that preferences are tilted towards the non-traded good leading to a share of non-traded or service consumption of $1 - \gamma = 0.4085$. For the share of home produced goods in the tradables basket we set $\upsilon = 0.52$ as in Natalucci and Ravenna (2003) who estimated the latter for the Czech economy. For the labour supplies to each sector we assume that $\frac{L_N}{L} = \frac{L_H}{L} = 0.5$.

As becomes visible of the log-linearised model to be shown in the appendix, dynamics depend on the initial asset position of the economy. From the External Sectors Indicators of the Balance of Payments statistics of the BNB we obtain the BNB foreign reserves to asset ratio.[15] We average the ratio for the time span $1999 - 2003$ which yields $\frac{Z}{F} = -0.9$ as the steady state value. Further, net external debt to GDP averaged over the same time span is 33.1%. In the absence of specific data we assume that home produced tradable GDP amounts to half of total GDP from which we can proxy that $\frac{F}{Y_H}$ is equal to -0.6. We set exports as share of net trade $\frac{C_H^*}{NX}$ to 3/2, thereby departing largely from the empirical average Bulgarian value given by -3.4 for the considered time span. Employing the value taken from the data would induce non-stationarity in the model, i.e. there would be no to return to the initial steady state after temporary shocks.[16] We set the time-invariant risk premium given by ξ equal to 5 percent.

Concerning domestic production, the quarterly depreciation rate of physical capital is set to the standard value of $\delta = 0.025$. As in Natalucci and Ravenna (2003) the elasticity of Tobin̉s Q with respect to the investment-capital ratio is 0.5 in both production sectors. We assume that home tradable production technology is twice as capital intensive as in the non-traded sector, i.e. $\alpha_H = 0.67$ and $\alpha_N = 0.33$.

The price setting-probabilities θ_N, θ_H and the share of backward looking firms ϖ_N, ϖ_H determine the characteristics of the inflation dynamics in the respective sector. Estimates of sector-specific hybrid inflation dynamics for Bulgaria are absent so far. We use - not-sector specific - estimates from Lendvai (2005) for the case of Hungary, which serve as a proxy for inflation dynamics in transition economies. The author finds the share of backward looking firms to be in the interval of $[0.3, 0.55]$, hence we set $\varpi_N = \varpi_H = 0.4$. Therefore, the share of forward-looking firms dominates in both sectors. The estimated probability in any period that a firm cannot reset its price is in the interval between 0.45 to 0.6, hence we assume $\theta_N = \theta_H = 0.55$ to be reasonable values. The setting implies that the average duration of price contracts - which is a measure of the nominal rigidity in each sector - for the non-traded sector becomes $D^N = \frac{1}{1-\varpi_N}\frac{1}{1-\theta_N} = 3.7$ quarters, the same in the traded goods sector. In other words, it takes around one year for prices to adjust to new information.

Estimates of equilibrium mark-ups in Bulgarian manufacturing sectors for the time-span $1995 - 2001$ are provided in Dobrinsky et al. (2004). We extract an average value around 1.2 and assume that the mark-up is same for both sectors. In other words, firms charge in equilibrium prices which are 20% above real marginal cost. From (55) and (56) we then obtain the price elasticities of demand $\rho_H = \rho_N = 6$.

Government consumption is set to 10 percent of sector-specific output.

[14] The detailed set of the calibrated parameters can be obtained from appendix A on page 26.

[15] BNB reserve assets (in mio Euro) / net external debt (in mio Euro), see BNB (2006).

[16] These assumptions have some implications for other shares in the external sector in long-run equilibrium. From (66) follows that the current account $NX + iF$ has to be balanced in steady state which requires $\frac{NX}{Y_H} = \frac{\beta-1}{\beta}\frac{F}{Y_H} = 0.0061$. For $\frac{C_H^*}{NX} = 3/2$ we obtain that $\frac{C_H^*}{Y_H} = \frac{C_H^*}{NX}\frac{NX}{Y_H} = \frac{3}{2}0.0061 = 0.0092$.

6 Economic Policy analyses

6.1 The Balassa-Samuelson effect under the Currency Board

The Balassa-Samuelson (BS) hypothesis suggests that higher factor productivity in tradables than non-tradables production - typical for the catch-up process - will contribute to an increase of inflation in non-tradables. [17] Under the currency board, the nominal exchange rate cannot adjust to absorb the pressure on the real exchange rate induced by the rise in traded-goods productivity causing domestic overall inflation rate to increase. To construct the BS-effect we assume that tradable factor productivity in the tradable sector $A_{H,t}$ grows in excess of $A_{N,t}$ till it reaches its steady state value $A_H = 1$. Non-traded sector productivity remains at its steady state value $A_N = 1$ throughout. For an autocorrelation coeffi cient in the total factor productivity process of $\rho_{Y_H} = 0.85$ we attain that after approximately 24 quarters, or 6 years A_H will have returned to its steady state value.[18]

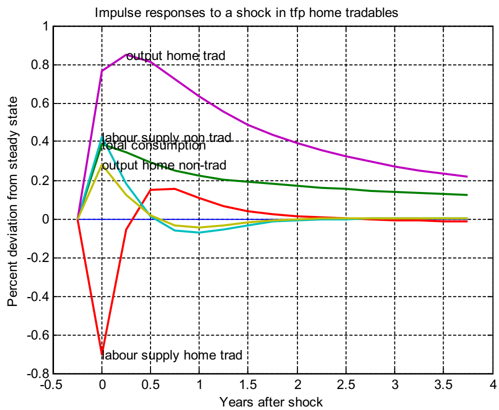

Figure 1: Real Sector

Figures 1 to 3 illustrate developments in the real, financial, and external sector following the BS shock. From 1 we see that there is persistent higher growth of real output in the tradable sector. The humped-shaped response can be attributed to the presence of costs in changing the initial stock of capital. The improved effi ciency of firms in the traded-goods sector coupled with fixed output demand due to the price rigidity results in initially less labour demand in the tradable sector. The higher productivity reduces real marginal cost in the tradable sector and it is profitable for firms which are able to reset prices to lower prices in order to sell more causing a drop in tradable inflation. Higher productivity allows for higher real wages in the H sector and due to labour mobility across the sectors real wages in the N sector will increase as well. The latter can only be accomplished by higher final goods prices in the non-tradable sector generating the initial surge in nontradable inflation. This effect becomes visible from figure 2 where the response in inflation is humped-shaped due to the presence of backward-looking firms in the N sector that cannot adjust price-setting immediately.

[17] Average productivity growth from 1999 − 2005 in Bulgaria was 5.70 for the tradables and 1.18 for the non-tradable sector (see Dimitrova 2006, forthcoming).

[18] As in (Natalucci and Ravenna, 2003,) no other shock affects the economy in subsequent periods and the initial shock at $t = 0$ generates the entire dynamics.

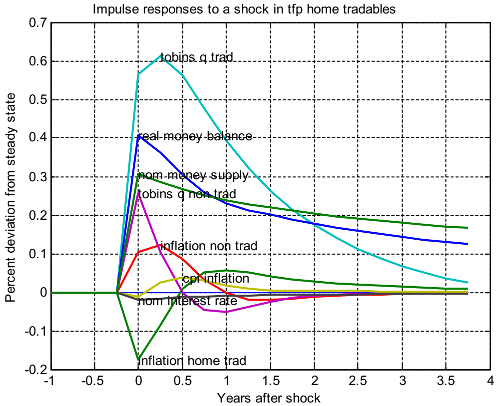

Figure 2: Financial sector

As increases in non-tradable inflation outweigh initial decreases in home-tradable inflation, there is a net increase in CPI inflation. The impact of the BS shock on CPI inflation is not that pronounced and starts dying out after 2 years as movements in H and N inflation rates start compensating for each other. The effect of the productivity gain in the H sector on the real exchange rate is more pronounced leading to a large persistent appreciation illustrated in figure 3. The result that the effect on the REER is way more persistent than on the domestic inflation rates can be explained by the increase in tradable consumption caused by higher tradable output.[19]

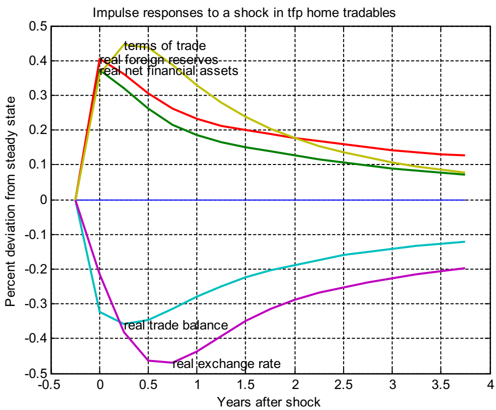

Figure 3: External Sector

[19]Note that the REER can be written as $q_t = c_{N,t} - c_{T,t}$. Following the BS-shock there is a persistent surge in tradable consumption.

The productivity shock improves the financial sector sentiment given by Tobin's Q in the respective sectors which becomes clear from figure 2. The improvement in the domestic financial market climate by the surge in the market value of capital over its replacement costs in the H sector improves the net asset position of the economy. The balance of payment turns temporarily into surplus by the respective increase in the net financial wealth as can be seen from figure 3. The accumulation of real foreign reserves causes a domestic monetary expansion which increases holdings of real cash balances and decreases the short-run interest rate, further contributing to the increase in total consumption. As there is a positive change in net foreign reserves, the current account is in surplus and wealth effects from factor income dominate the visible decrease in the real trade balance.

Concerning compliance with the EU Accession Requirements, we obtain that for the given calibration there would be no harm done by the BS effect regarding the fulfillment of the Maastricht inflation rate and short-term interest rate criteria. Our model therefore gives some intuition for the empirically acknowledged fact that the contribution of the BS effect to domestic inflation is not essential (see Dimitrova 2006 for the empirical evidence). In related research, Natalucci and Ravenna (2003) find that non-currency board fixed exchange rate accession countries that are exposed to a BS type shock face high inflation in the non-tradable sector that pushes CPI inflation up, well beyond the Maastricht limit. We therefore obtain evidence that inflationary and interest rate perfomance under the currency board is superior to the fixed exchange rate regime. However, without a richer fiscal side featuring the possibility of fiscal deficits and the build-up of government debt we cannot draw any further conclusions from our structural framework relating to this point.

6.2 Real Convergence and the Real Exchange Rate in the Long-Run

In order to investigate long-run sustainability issues we expose our model to a permanent TFP shock in the H sector. From figure 4 we see that in the long-run, the internal real exchange rate appreciation is pronounced. The relative price of non-tradables increases by around 7% in the long run and is accompanied by a transitory and decreasing improvement in the real net financial asset position. Households therefore increase their tradable and non-tradable consumption by around 7% and 4% respectively by de-accumulating their lifetime wealth. Eventually, higher overall consumption at steady state stems solemnly from higher steady state output made possible by higher factor productivity in the H sector.

6.3 Fiscal Policies and the Trade Balance

Turning to the demand side we investigate the impact of a transitory increase in government spending in the N and H sector, respectively. The standard flex-price RBC closed-economy framework predicts a decline in total consumption in response to a rise in government spending. Ricardian consumers consume out of life-time wealth and they smooth away income fluctuations caused by the initial surge in government spending. This result is also visible in our sticky-price open economy model as becomes clear from figures 5 and 6. We further can extract the impact of government spending on the real net foreign asset position of the economy.

Turning to figure 5 we see that a temporary fiscal policy shock increases home tradable inflation and appreciates the terms of trade causing a loss in external competitiveness. The higher inflation triggered by demand side pressures contracts the real domestic base money supply which under the currency board mechanism leads to a one-for-one loss in real reserve holdings. De-accumulation in reserves results in a deterioration of the intertemporal foreign asset position. Households foresee their loss in life-time wealth and smooth the effect away by initially decreasing total consumption. Following the shock, home tradable output increases and as domestic absorbtion by households decreases, a positive net trade balance builds up.[20]

[20] The impulse response of sector-specific output and total consumption is omitted from both graphs to enhance

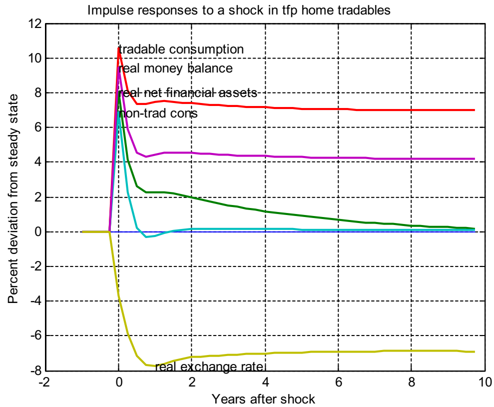

Figure 4: Long run development of the REER and the external asset position

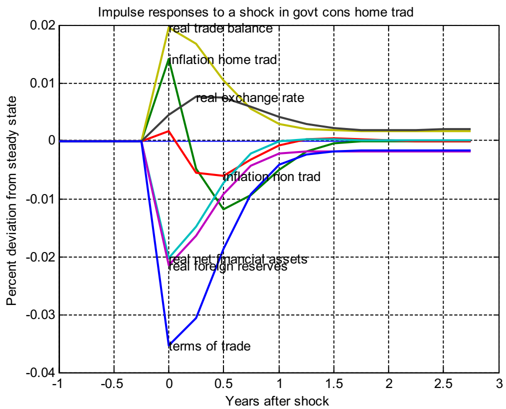

Figure 5: External sector after a persistent increase in government tradable consumption

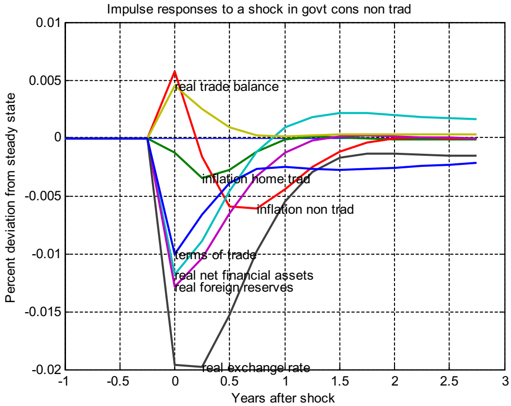

Figure 6: External sector after a persistent increase in government non-tradable consumption

As there is an overall loss in reserves, the intertemporal loss in net wealth of the economy dominates.

Turning to figure 6, we see that mainly the same qualitative conclusions arise if government expenditures fall on services, i.e. N consumption. The single main difference lies in the development in the real exchange rate. The initial surge in spending causes an increase in non-tradable inflation and hence to a rise in the relative price of non-tradables. We see that the real exchange rate appreciation is less persistent than the development in the home non-tradable inflation. It hence turns out that government expenditures that fall on non-traded goods rather than traded goods contribute to the worsening in external competitiveness triggered by the BS effect.

7 Conclusions

The paper investigated the transmission of productivity and fiscal shocks under the currency board mechanism within a New Open Economy Macroeconomic model calibrated to the Bulgarian economy. We found that inflationary pressures resulting from real catch-up processes will not have an persistent impact on overall CPI inflation and will therefore not prevent compliance with the Maastricht inflation criterion. Nevertheless a persistent appreciation in the real exchange rate is visible. Fiscal spending in either sector has a positive short run impact on the trade balance but still leads to current account deficits due to the long-run worsening of the net foreign asset position. When fiscal spending falls on the traded (non-traded) goods, a lasting real depreciation (appreciation) of the real exchange rate shows up.

For a full assessment of the readiness of the Bulgarian economy for soon ERM II and Euro-area accession entry more work needs to be done on the fiscal sector of the model which might affect the transmission of shocks. Introduction of a proportion of non-Ricardian households who consume out of current instead of permanent income can lead to different results regarding the effects of fiscal spending. Monetary stabilisation policies made possible by drawing from government deposits in the central bank have to be investigated. The current results might also be challenged if different degrees of pass-through of foreign to domestic inflation are introduced.

clarity of the exposition.

Eventually, the role of foreign direct investment as intermediate good in tradable goods production has to be studied. Nevertheless the model set up here can serve as a first step towards a fuller treatment of analysing Bulgaria's rapprochement to the Euro-zone within the framework of the New Open Economy Macroeconomics.

Technical Appendices

A Calibration

Parameters calibrated for Quarterly Data

Description	Parameter	Value	Source
risk premium	ξ	0.05	
share of home-trad in trad basket	ν	0.52	Natalucci and Ravenna (2003)
share of trad in consum basket	γ	0.5915	Valev (2005)
inverse of elasticity of labour supply	κ	2	Natalucci and Ravenna (2003)
depreciation rate of physical capital	δ	0.025	RBC
demand elasticity of H consumption	ρ_H	6	Dobrinsky et al. (2004)
demand elasticity of N consumption	ρ_N	6	Dobrinsky et al. (2004)
households' discount factor	β	0.99	RBC
share of capital in home trad production	α_H	0.67	Natalucci and Ravenna (2003)
share of capital in non-trad production	α_N	0.33	Natalucci and Ravenna (2003)
elasticity of investment-capital ratio	η	0.5	Natalucci and Ravenna (2003)
share of backw-looking firms in H sector	ϖ_H	0.4	Lendvai (2005)
share of backw-looking firms in N sector	ϖ_N	0.4	Lendvai (2005)
price resetting probability in H sector	$1-\theta_H$	0.45	Lendvai (2005)
price resetting probability in N sector	$1-\theta_N$	0.45	Lendvai (2005)
ut. scale parameter real money balances	χ	0.005	Henriksson (2005)
steady state ratio reserves to assets	$\frac{Z}{F}$	-0.9	BNB (2006)
steady st share of labour supply in H sector	$\frac{L_H}{L}$	0.5	
steady st share govt H consumption	$\frac{G_H}{Y_H}$	0.1	Natalucci and Ravenna (2003)
steady st share govt N consumption	$\frac{G_N}{Y_N}$	0.1	Natalucci and Ravenna (2003)
steady st share exports to net trade balance	$\frac{C_H^*}{NX}$	3/2	
coeffi cient H productivity process	ρ_{Y_H}	$\{0.85, 1\}$	Natalucci and Ravenna (2003)
coeffi cient N productivity process	ρ_{Y_N}	$\{0.8, 1\}$	Natalucci and Ravenna (2003)
coeffi cient H tradables govt. consumption	ρ_{G_H}	$\{0.43, 1\}$	Natalucci and Ravenna (2003)
coeffi cient N non-trad govt. consumption	ρ_{G_N}	$\{0.43, 1\}$	Natalucci and Ravenna (2003)
coeffi cient external demand process	$\rho_{C_H^*}$	$\{0.3, 1\}$	
coeffi cient EMU interest rate process	ρ_{i^*}	$\{0.9, 1\}$	Natalucci and Ravenna (2003)

Implied Steady State Values

Description	Parameter	Value
Home Consumption to Output H sector	$\frac{C_H}{Y_H}$	0.49324
Home Consumption to Output N sector	$\frac{C_N}{Y_N}$	0.70414
Investment to Output H sector	$\frac{I_H}{Y_H}$	0.39766
Investment to Output N sector	$\frac{I_N}{Y_N}$	0.19586
steady state mark-up H sector	$(\frac{\rho_H - 1}{\rho_H})^{-1}$	1.2
steady state mark-up N sector	$(\frac{\rho_N - 1}{\rho_N})^{-1}$	1.2
Capital to Output ratio H sector	$\frac{K_H}{Y_H}$	15.906
Capital to Output ratio N sector	$\frac{K_N}{Y_N}$	7.8345
Exports to home produced goods	$\frac{C_H^*}{Y_H}$	0.0091

B Exogenous stochastic processes

For sector specific total factor productivity (TFP) of the H sector we assume

$$A_{H,t} = \exp[\rho_{Y_H} a_{H,t-1} + \varepsilon_{Y_H,t}]$$

Analogously for the N sector TFP

$$A_{N,t} = \exp[\rho_{Y_N} a_{N,t-1} + \varepsilon_{Y_N,t}]$$

Government demand comes from

$$\begin{aligned} G_{H,t} &= \exp[\rho_{G_H} a_{G_H,t-1} + \varepsilon_{G_H,t}] \\ G_{N,t} &= \exp[\rho_{G_N} a_{G_N,t-1} + \varepsilon_{G_N,t}] \end{aligned}$$

And external demand follows

$$C^*_{H,t} = \exp[\rho_{C^*_H} a_{C^*_H,t-1} + \varepsilon_{C^*_H,t}]$$

The error terms in each process have log-normal nid(0,1) distributions.

C The log-linearised model

The log-linearised model is given by the following set of equations, where we have already used that under the currency board $s_t = 0$ at all dates. We can reduce the number of variables by substituting out all price level variables, i.e. cpi price level p_t, home traded price level $p_{H,t}$, non-traded price level $p_{N,t}$ as well as traded goods price level $p_{T,t}$. Note that $E_t p_{t+1} - p_t = E_t \pi_{t+1}$, analogously for H, N and T.

Non-Expectational Equations

$$
\begin{aligned}
0 &= c_t - \gamma c_{T,t} - (1-\gamma)c_{N,t} \\
0 &= c_{T,t} - c_{N,t} - \frac{1}{1-\gamma}i_t + \frac{1}{1-\gamma}c_t - \frac{1}{1-\gamma}m_t + (v + \frac{\gamma v}{1-\gamma})c_{F,t} - (v + \frac{\gamma v}{1-\gamma})c_{H,t} \\
0 &= y_{H,t} - a_t^H - \alpha_H k_{t-1}^H - (1-\alpha_H)l_t^H \\
0 &= y_{N,t} - a_t^N - \alpha_N k_{t-1}^N - (1-\alpha_N)l_t^N \\
0 &= y_{H,t} - \frac{C_H}{Y_H}c_{H,t} - \frac{I_H}{Y_H}i_{H,t} - \frac{G_H}{Y_H}g_{H,t} - \frac{C_H^*}{Y_H}c_{H^*,t} \\
0 &= y_{N,t} - \frac{C_N}{Y_N}c_{N,t} - \frac{I_N}{Y_N}i_{N,t} - \frac{G_N}{Y_N}g_{N,t} \\
0 &= l_t - \frac{L^N}{L}l_t^N - \frac{L^H}{L}l_t^H \\
0 &= w_t - i_t - m_t - (\kappa - 1)l_t \\
0 &= w_t^r - w_t + i_t - c_t + m_t \\
0 &= k_t^H - \delta i_t^H - (1-\delta)k_{t-1}^H \\
0 &= k_t^N - \delta i_t^N - (1-\delta)k_{t-1}^N \\
0 &= q_t^H - \eta i_t^H + \eta k_{t-1}^H \\
0 &= q_t^N - \eta i_t^N + \eta k_{t-1}^N \\
0 &= \hat{\lambda}_t^C + \pi_{H,t} - \pi_t + (1-\beta(1-\delta))r_t^H + \beta q_t^H - \hat{\lambda}_{t-1}^C - q_{t-1}^H \\
0 &= \hat{\lambda}_t^C + \pi_{N,t} - \pi_t + (1-\beta(1-\delta))r_t^N + \beta q_t^N - \hat{\lambda}_{t-1}^C - q_{t-1}^N \\
0 &= \hat{\lambda}_t^C + c_t \\
0 &= r_t^H - mc_t^H - y_t^H + k_{t-1}^H \\
0 &= r_t^N - mc_t^N - y_t^N + k_{t-1}^N \\
0 &= \pi_t - \gamma \pi_{T,t} - (1-\gamma)\pi_{N,t} \\
0 &= \pi_{T,t} - v\pi_{H,t} \\
0 &= \hat{\imath}_t^* - \hat{\imath}_t \\
0 &= \hat{\imath}^* - \widehat{\hat{\imath}^*} + \xi f_t^r \\
0 &= f_t^r - \frac{1}{\beta}f_{t-1}^r - \frac{1}{\beta}\hat{\imath}_{t-1} + (\frac{1}{\beta} + \frac{Z}{F})\pi_t + \frac{1-\beta}{\beta}nx_t - \frac{(1-\beta)(\gamma-1)}{\beta}q_t + \frac{(1-\beta)v}{\beta}t_t + \frac{Z}{F}z_t^r - \frac{Z}{F}z_{t-1}^r \\
0 &= f_t - f_t^r - i_t + c_t - m_t \\
0 &= nx_t - \frac{C_H^*}{NX}c_{H^*,t} + \frac{C_H^*}{NX}c_{H,t} + (\frac{C_F}{NX} - \frac{C_H^*}{NX})c_{F,t} \\
0 &= nx_t^r - nx_t - t_t \\
0 &= q_t + c_{T,t} - c_{N,t} \\
0 &= t_t + c_{F,t} - c_{H,t} \\
0 &= m_t^r + i_t - c_t \\
0 &= m_t - z_t
\end{aligned}
$$

Expectational equations

$$\begin{aligned}
0 &= E_t c_{t+1} - c_t - \hat{\imath}_t + E_t p_{t+1} - p_t \\
0 &= E_t c_{H,t+1} - c_{H,t} - \hat{\imath}_t + E_t p_{H,t+1} - p_{H,t} \\
0 &= E_t c_{N,t+1} - c_{N,t} - \hat{\imath}_t + E_t p_{N,t+1} - p_{N,t} \\
0 &= -\lambda_H^f E_t \pi_{H,t+1} + \pi_{H,t} - \lambda_H^{mc} mc_{H,t} - \lambda_H^b \pi_{H,t-1} \\
0 &= -\lambda_N^f E_t \pi_{N,t+1} + \pi_{N,t} - \lambda_N^{mc} mc_{N,t} - \lambda_N^b \pi_{N,t-1}
\end{aligned}$$

Exogenous stochastic processes

$$\begin{aligned}
E_t a_{H,t+1} &= \rho_{Y_H} a_{H,t} + E_t \varepsilon_{Y_H,t+1} \\
E_t a_{N,t+1} &= \rho_{Y_N} a_{N,t} + E_t \varepsilon_{Y_N,t+1} \\
E_t g_{H,t+1} &= \rho_{G_H} g_{H,t} + E_t \varepsilon_{G_H,t+1} \\
E_t g_{N,t+1} &= \rho_{G_N} g_{N,t} + E_t \varepsilon_{G_N,t+1} \\
E_t c^*_{H,t+1} &= \rho_{C^*_H} c^*_{H,t} + E_t \varepsilon_{C^*_H,t+1} \\
E_t i^*_{t+1} &= \rho_{i^*} i_t + E_t \varepsilon_{i^*,t+1}
\end{aligned}$$

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