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DP/63/2008

A Small Open Economy Model with a Currency Board Feature: the Case of Bulgaria

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January 2008

ISBN: 978–954–8579–11–7

Printed in BNB Printing Centre.

Views expressed in materials are those of the authors and do not necessarily reflect BNB policy.

Elements of the 1999 banknote with a nominal value of 50 levs are used in cover design.

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SUMMARY. The paper presents a first version of a dynamic stochastic general eqiulibrium model for a small open economy operating under a currency board arrangement. The model specification and calibration used are tailored to the case of the Bulgarian economy. We discuss the model solution, describe the results of several simulation scenarios and identify directions for extending and improving the model.

Key words: DSGE, small open economy, currency board.

The bulk of this paper was completed while Iordan Iordanov was visiting researcher at the Bulgarian National Bank. The authors would like to thank Mariela Nenova, Rossen Rozenov and participants in an internal seminar at the BNB for useful comments and suggestions. The usual disclaimer applies.

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1 Introduction

Since the 1980s there has been a marked shift of academic research in macroeconomics towards dynamic models with explicit microeconomic foundations. The potential of such models to combine different groups of economic agents with well-articulated motivation under a coherent stock-flow framework has made them the tool of choice for investigating a number of important economic issues and thus the dynamic stochastic general equilibrium (DSGE) paradigm was born. This has not gone unnoticed by applied researchers and, in particular, many central banks have recently begun to expand their analytical toolkits to include DSGE models.

The sound theoretical foundations of DSGE models make them potentially suitable for cases when short time series and noisy data limit the applicability of more traditional models based on econometrically estimated relationships. Thus the DSGE approach provides an alternative for modelling in transition economies and offers the possibility to explore scenarios that are not amenable to analysis with classical macroeconometric models. A number of questions pertaining to the sustainability of different economic situations or the response of the economy to various disturbances can be fruitfully analyzed in a DSGE framework. Indeed, the present work started out as a project to study current account dynamics in the case of an economy which – like Bulgaria – operates under a currency board arrangement, although the scope of the analysis was expanded as the work progressed.

The initial inspiration for the modelling framework we have chosen can be traced back to the new open economy literature that emerged in the 1990s. Obstfeld and Rogoff (1996) deals in detail with the analysis of small open economies. The current account balance plays a key role in describing the interaction of a small open economy with the rest of the world. Chapter I of the book is mainly devoted to exploring the concept of the current account and tying it together with various entities commonly looked at in economic analyses, such as GDP, consumption, investment and assets. In the very beginning of the book (p.8), a simple model is developed to support the claim that an unbalanced current account is not necessarily bad for the economy.

A relatively short survey of the different approaches for studying and modelling the current account is offered in Obstfeld and Rogoff (1994), where the issues are copiously illustrated with data for different countries over long time spans. The authors start out by recalling the Lucas critique and emphasizes the advantages of macroeconomic models based on optimal microeconomic behaviour over purely econometric models whose properties are essentially data-driven. The survey offers an overview of a number of models that emphasize different economic mechanisms, such as the single-good model with a representative resident, models with investment and cost of installing, non-tradable goods, durable goods, the effects of terms of trade and international transfers, as well as demographic structure and fiscal policies.

The classification of stochastic models in Obstfeld and Rogoff (1994) is performed according to the structure of financial markets they employ. Markets can be complete (§3.2.1) or, in a simpler setup, can feature only trade in bonds (§3.2.2). Partially complete markets are also possible (§3.2.3). The survey of the various alternatives available leads the authors to the conclusion that one needs to strive toward building "richer and more realistic intertemporal models" featuring the interaction of diversified economic mechanisms.

Motivated by the above considerations, in this paper we develop a DSGE model for a small open economy that operates under a currency board arrangement, specifically taking into account the characteristics of the Bulgarian economy. In our approach we try to isolate the most prominent mechanisms at work in the economy, aiming at parsimony rather than comprehensiveness. Compared to earlier attempts to study certain features of the Bulgarian economy in a general equilibrium framework, such as Blessing (2007) and Valev (2006), this paper models the currency board arrangement under more flexible assumptions and explicitly features an active government pursuing independent objectives.

The paper is organized in the following way. Section 2 describes the model. Section 3 spells out the derivation of the necessary conditions and the steady state. Details of the model calibration and solution are provided in section 4. We present the results of some simulations in section 5. Section 6 concludes and provides some directions for improving and extending the model.

2 The model

The model features two types of production – production of tradable and of non-tradable goods – and a household sector. In addition, there is a government that dynamically optimizes its own utility function and a central bank operating under a currency board arrangement, effectively modelled as an identity in the model.

In our setup there is a continuum of households indexed on the interval $[0,1]$. A similar structure is assumed for the sector producing non-tradable goods. The problem for household z is described in subsection 2.1 below.

2.1 Household z

The problem for household z is a modified version of the one in Obstfeld and Rogoff (1995). Household z maximizes the utility functional

$$\begin{aligned} U(z) =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\gamma \ln(C_{Tt} - b_T C_{Tt-1}) + (1-\gamma) \ln(C_{Nt} - b_N C_{Nt-1}) \\ & + \frac{\chi}{1-\epsilon} \left(\frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\kappa}{2} (L_{Tt}(z) + L_{Nt}(z))^2 \Bigg], \end{aligned} \tag{1}$$

where C_T is consumption of tradable goods [1], C_N is consumption of non-tradable goods, M is nominal money balances, P is the overall price level and $L_T(z)$ ($L_N(z)$) stands for the labour input of household z in tradable (non-tradable) sector production. The specification includes habit formation mechanisms in consumption, as captured by the coefficients b_T and b_N. The disutility from labour enters symmetrically to reflect the assumption that, other things equal, households should be indifferent between supplying a unit of labour to the tradable vs. the non-tradable sector and thus should base their decision on the respective wage earned. As in Erceg, Henderson and Levin (2000), our specification omits the indexing of some variables by the household-specific index z, which is a consequence of assuming complete contingent claims markets for the respective quantities.

Household z has a net real asset position F which evolve according to the following equation:

$$\begin{aligned} P_{Tt} F_{t+1} =& P_{Tt}(1+i_t) F_t - Z_t + W_{Tt}(z) L_{Tt}(z) + W_{Nt}(z) L_{Nt}(z) - \\ & - P_{Tt} C_{Tt} - P_{Nt} C_{Nt} + \Pi_t(z) + \Pi_{Tt} - P_t T_t. \end{aligned} \tag{2}$$

Here i denotes the interest rate on the asset position of the household. Following Schmitt-Grohé and Uribe (2003), it is given by the international interest rate i^* plus a risk premium depending on the net asset position relative to a perceived equilibrium asset position $\bar{F}$:

$$i_t = i_t^* + p(F_t) = i_t^* + \psi(e^{\bar{F} - F_t} - 1). \tag{3}$$

[1] To facilitate the exposition we omit the time subscripts on the variables whenever no confusion arises from doing so.

The variables $W_T(z)$ and $W_N(z)$ stand for the wage rates for household z in the tradable and non-tradable sectors, respectively. The profits of the firms in the tradable and non-tradable sectors, respectively Π_T and $\Pi(z)$, are transferred to the household as dividends each period (see below). It should be noted that although this formulation links the transfer of profits of firm z specifically to the z-th household, standard symmetry assumptions as employed in the literature rule out any special consequences of this choice. The variable T denotes real lump-sum taxes collected by the government.

Purchasing power parity holds for the price of tradable goods, P_T, which depends on the exchange rate E and the international price of tradable goods P_T^* according to equation (4).

$$P_{Tt} = E_t P_{Tt}^*. \tag{4}$$

For the purposes of this work, the exchange rate may be interpreted in two ways. First, it is possible to interpret E as a nominal effective exchange rate, so that the peg to the reserve currency (which presumably accounts for a large share of foreign trade) provides the level E_L, while fluctuations around it come from other currencies used in foreign trade transactions. Second, one may prefer to assume that all foreign trade takes place at the fixed exchange rate, in which case we shut down the stochastic component by setting to zero the variance σ_E^2 in (6) below. Formally, we model the exchange rate as randomly fluctuating around a given level E_L (see equation (5)) according to a stochastic process $\bar{E}_t$ as specified in equation (6). This specification, in view of the currency board arrangement, provides us with flexibility to adopt the alternative interpretations.

$$E_t = E_L + \bar{E}_t. \tag{5}$$

$$\bar{E}_{t+1} = \rho_E \bar{E}_t + \varepsilon_{Et+1},\ 0 < \rho_E < 1,\ \varepsilon_{Et} \sim iid\ N(0, \sigma_E^2) \tag{6}$$

Similarly, we allow for stochastic fluctuations in the international price of tradable goods, P_{Tt}^*, as specified in equation (7), where l_{PT}^* is a scaling factor.

$$\begin{gathered} P_{Tt}^* = l_{PT}^* e^{\bar{P}_T^*},\ \bar{P}_{Tt+1}^* = \rho_{PT} \bar{P}_{Tt}^* + \varepsilon_{P_T^* t+1}, \\ 0 < \rho_{PT} < 1,\ \varepsilon_{P_T^* t} \sim iid\ N(0, \sigma_{P_T^*}^2) \end{gathered} \tag{7}$$

The overall price index is defined as a weighted average of the prices of tradable and non-tradable goods:

$$P_t := \gamma P_{Tt} + (1 - \gamma) P_{Nt} \tag{8}$$

Money holdings evolve according to equation (9), with Z being the change in money holdings from one period to the next.

$$M_{t+1} = M_t + Z_t. \tag{9}$$

The household maximizes utility subject to equations (2) and (9) by choosing at each moment the variables W_T, W_N, C_T, C_N and Z. The choice of wages by the household is based on Christiano, Eichenbaum and Evans (2005), which allows room to include wage rigidity in the model.

2.2 Firm z in the non-tradable sector

Similarly to households, firms in the non-tradable sector also operate in a monopolistically competitive environment. Firm z seeks to maximize its value

$$\begin{aligned} V(z) =&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big\{ p_{Nt}(z) y_{Nt}(z) - W_{Nt}(z) L_{Nt}(z) - \\ &- p_{Nt}(z) I_{Nt}(z) - P_{Ot} O_{Nt}(z) \Big\}. \end{aligned} \tag{10}$$

subject to technological and capital accumulation constraints as specified below. The variables $p_N(z)$ and $y_N(z)$ are respectively the price and the quantity produced of the z-th variety of the non-tradable good, $I_N(z)$ stands for investment by firm z, $O_N(z)$ is the energy resource input used by the firm in production and P_O is the domestic price of energy resources. Energy resources are not produced locally and therefore can only be imported. In line with much of the literature, the firm discounts the future using the discount factor $\beta^t \mu_t$, where μ_t is the shadow price on the household's resource constraint. This ensures consistency between the intertemporal preferences of the firm and its shareholders.

The production technology of firm z is given by a three-factor Cobb-Douglas production function exhibiting constant returns to scale, with energy resources as the third input:

$$y_{Nt}(z) = A_{Nt} L_{Nt}(z)^{\alpha_N} K_{Nt}(z)^{\beta_N} O_{Nt}(z)^{1-\alpha_N-\beta_N}. \tag{11}$$

The evolution of productivity in the non-tradable sector is the same across firms. It is given by

$$\begin{aligned} A_{Nt} = e^{\bar{A}_{Nt}},\ \bar{A}_{Nt+1} = \rho_N \bar{A}_{Nt} + \varepsilon_{Nt+1}, \\ 0 < \rho_N < 1,\ \varepsilon_{Nt} \sim iid\ N(0, \sigma_N^2). \end{aligned} \tag{12}$$

Analogously to the determination of tradable goods prices, the domestic price of energy resources emerges through a PPP-type mechanism:

$$P_{Ot} = E_t P^*_{Ot}. \tag{13}$$

In turn, the process for the external energy resource price is

$$\begin{gathered} P^*_{Ot} = l^*_{PO} e^{\bar{P}^*_{Ot}},\ \bar{P}^*_{Ot+1} = \rho_{PO} \bar{P}^*_{Ot} + \varepsilon_{P_O t+1}, \\ 0 < \rho_{PO} < 1,\ \varepsilon_{P_O t} \sim iid\ N(0, \sigma^2_{P_O}). \end{gathered} \tag{14}$$

The firm accumulates capital in a standard manner as given by equation (15).

$$K_{Nt+1}(z) = (1-\delta) K_{Nt}(z) + I_{Nt}(z). \tag{15}$$

The firm maximizes its value by choosing $L_N(z)$, $I_N(z)$ and $O_N(z)$ each period. Firm z's profit is

$$\Pi_t(z) = p_{Nt}(z) y_{Nt}(z) - W_{Nt}(z) L_{Nt}(z) - p_{Nt}(z) I_{Nt}(z) - P_{Ot} O_{Nt}(z) \tag{16}$$

and is transferred to household z.

2.3 Aggregating firm in the non-tradable sector

Specific varieties of the goods produced in the non-tradable sector are bundled together by a representative aggregating firm. The production technology employed by the firm is given by

$$y_{Nt} = \left[\int_0^1 y_{Nt}(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \tag{17}$$

For accounting consistency, we require that the total output of non-tradable goods equals aggregate demand for non-tradables:

$$y_{Nt} = C^A_{Nt} + G_{Nt} + I^A_{Nt}. \tag{18}$$

The aggregating firm chooses the amount of each input $y_N(z)$ in the bundle.

In a standard manner one can derive equations (19) for the aggregate price of non-tradables and (20) for the demand for product z from the optimization problem for this sector.

$$P_N = \left[\int_0^1 p_N(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{19}$$

$$y^d_{Nt}(z) = \left[\frac{p_{Nt}(z)}{P_{Nt}} \right]^{-\theta} y_{Nt}. \tag{20}$$

2.4 Labour-aggregating firm for $L_{Tt}(z)$

Similarly to the setup for the aggregating firm in the non-tradable sector, the labour inputs from the households to the tradable sector are aggregated by a firm that uses the technology

$$L_{Tt} = \left[\int_0^1 L_{Tt}(z)^{\frac{\lambda-1}{\lambda}} dz \right]^{\frac{\lambda}{\lambda-1}} \tag{21}$$

and chooses the amount of each input $L_T(z)$.

One can obtain the demand for labour type z and the average wage in the tradable sector from this optimization problem (see equations (22) and (23), respectively).

$$L^d_{Tt}(z) = \left[\frac{W_{Tt}(z)}{W_{Tt}} \right]^{-\lambda} L_{Tt}. \tag{22}$$

$$W_T = \left[\int_0^1 W_T(z)^{1-\lambda} dz \right]^{\frac{1}{1-\lambda}}. \tag{23}$$

2.5 Representative firm in the tradable sector

A continuum of identical firms populate the tradable sector. They operate in an "almost-competitive" environment in the sense that they face the same price conditions at home and on international markets by virtue of (4), as well as unlimited external demand. We assume that firms in the sector always satisfy home orders first and export the balance. (Conversely, if home orders for tradable goods exceed output, the remainder is automatically imported.) We depart from the competitive paradigm by assuming that the market for tradable goods is not contestable. This allows firms to retain a positive level of profit in equilibrium, which is convenient from an empirical perspective.

The tradable sector representative firm seeks to maximize its value

$$V_T = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left\{ P_{Tt} y_{Tt} - W_{Tt} L_{Tt} - P_{Tt} I_{Tt} - P_{Ot} O_{Tt} \right\}, \tag{24}$$

subject to the usual constraints as described below. The notation for this sector conforms, with obvious modifications, to the conventions employed in the description of the non-tradable sector.

Capital evolves according to the equation

$$K_{Tt+1} = (1-\delta) K_{Tt} + I_{Tt}. \tag{25}$$

The production technology in the tradable sector mirrors the structure of that in the non-tradable sector and is given by

$$y_{Tt} = A_{Tt} L_{Tt}^{\alpha_T} K_{Tt}^{\beta_T} O_{Tt}^{1-\alpha_T-\beta_T}. \tag{26}$$

The technology process is

$$\begin{aligned} A_{Tt} = e^{\bar{A}_{Tt}},\ \bar{A}_{Tt+1} = \rho_T \bar{A}_{Tt} + \varepsilon_{Tt+1}, \\ 0 < \rho_T < 1,\ \varepsilon_{Tt} \sim iid\ N(0, \sigma_T^2). \end{aligned} \tag{27}$$

Each period the representative firm in the tradable sector chooses L_T, I_T and O_T in order to solve its maximization problem. Its per-period profit is given by

$$\Pi_{Tt} = P_{Tt} y_{Tt} - W_{Tt} L_{Tt} - P_{Tt} I_{Nt} - P_{Ot} O_{Tt}, \tag{28}$$

and is transferred to the households as dividend. With a continuum of firms in the sector and a continuum of households there exists a one-to-one mapping between each tradable sector firm and each household. Since firms are identical, we can effectively decouple the firm-household pairs and add the tradable-sector firm profit in the household resource constraint without indexing it.

2.6 Government

The treatment of the government in our model differs from the most common setups in the literature, where the government is either largely taken as an exogenous entity following, for instance, a balanced budget policy, or else is markedly benevolent and tries to maximize some measure of social welfare. The first route could be problematic when modelling an economy operating under a currency board arrangement, where by the very construction of the monetary regime the government is given a more prominent role in that fiscal policy is practically the sole instrument at the disposal of policymakers. The second route is more appealing, since it provides a clear motivation for government actions. As a simpler starting benchmark, however, we model the government as maximizing an objective function combining government spending, valued positively, and (lump-sum) taxes, valued negatively. This setup can serve as a first step toward introducing a welfare-maximizing government in the model, assuming that government spending goes mainly to the provision of public goods, thus ensuring the smooth functioning of the economic system. As we shall see, such an approximation is in certain respects crude and can lead to peculiarities in the government behaviour.

More specifically, the government's objective function is

$$U^G = \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\varkappa \ln G_{Tt} + (1-\varkappa) \ln G_{Nt} - \frac{\eta}{2} T_t^2 \right], \tag{29}$$

where δ is a time-discount factor, G_T stands for real government consumption of tradable goods and G_N is real government consumption of non-tradable goods. The parameters $\varkappa$ and η account for the relative importance of taxes and consumption for the government.

The currency board feature is captured by means of a stylized version of the Issue Department[2] balance:

$$P_{Tt} F_t^* = M_t + \Gamma_t. \tag{30}$$

Here Γ is the government deposit held with the central bank and $P_T F^*$ is the nominal value of foreign exchange reserves. Recall that the nominal currency stock M is determined by the household's problem.

The primary budget balance is given by the equation

$$P_{Tt} G_{Tt} + P_{Nt} G_{Nt} = P_t T_t + D_t, \tag{31}$$

[2] Organizationally, the Bulgarian National Bank is divided into several departments. One of them, the Issue Department, carries out the functions of a currency board.

where D_t denotes the primary deficit. Then the government deposit evolves according to

$$\Gamma_{t+1} = \Gamma_t + r^* P_{Tt} F_t^* - D_t = (1 + r^*)\Gamma_t + r^* M_t - D_t, \tag{32}$$

with r^* being the interest rate earned on foreign exchange reserves. Equation (32) implies that the central bank's profit is transferred to the government. In this model, we assume away the possibility of the government having or issuing debt, so that the primary balance is directly added to or subtracted from the government deposit.

The government seeks to maximize (29) subject to (32) by choosing paths for G_T, G_N and T.

To facilitate the analysis we define several important concepts – the cash balance

$$CB_t := P_t T_t - P_{Tt} G_{Tt} - P_{Nt} G_{Nt} + r^*(M_t + \Gamma_t), \tag{33}$$

the current account

$$\begin{aligned} CA_t :=& P_{Tt} y_{Tt} - P_{Tt} C_{Tt} - P_{Tt} G_{Tt} - P_{Tt} I_{Tt} - \\ & - P_{Ot}(O_{Tt} + O_{Nt}) + P_{Tt} i_t F_t + r^*(M_t + \Gamma_t), \end{aligned} \tag{34}$$

the total wagebill for the economy

$$WB_t := L_{Tt} W_{Tt} + L_{Nt} W_{Nt} \tag{35}$$

and the total employment

$$L_t := L_{Tt} + L_{Nt} \tag{36}$$

– which we track in the simulations below.

3 Necessary conditions, model closure and steady state calculation

In providing the necessary conditions for optimality for the various problems described above, we follow the convention adopted in Bodart et al. (2006) (see their footnote on p.9) and omit the conditional expectation operators from the equations that follow. Nonetheless, all the relationships should be interpreted as holding in expectation.

The necessary conditions for the consumer are

$$(1-\gamma)\frac{\Delta_{Nt}}{P_{Nt}} = \gamma\frac{\Delta_{Tt}}{P_{Tt}}, \tag{37}$$

$$\Delta_{Nt} := \frac{1}{C_{Nt} - b_N C_{Nt-1}} - \frac{\beta b_N}{C_{Nt+1} - b_N C_{Nt}},$$
$$\Delta_{Tt} := \frac{1}{C_{Tt} - b_T C_{Tt-1}} - \frac{\beta b_T}{C_{Tt+1} - b_T C_{Tt}},$$

$$\beta\Delta_{Tt}(1 + i_t - \psi F_t e^{\bar{F} - F_t}) = \Delta_{Tt-1}, \tag{38}$$

$$\beta\chi\left(\frac{M_t}{P_t}\right)^{-\epsilon}\frac{1}{P_t} + \beta\gamma\frac{\Delta_{Tt}}{P_{Tt}} = \gamma\frac{\Delta_{Tt-1}}{P_{Tt-1}}, \tag{39}$$

$$\kappa\lambda(L_{Tt}(z) + L_{Nt}(z)) = \gamma(\lambda - 1)\frac{\Delta_{Tt}}{P_{Tt}}W_{Tt}(z), \tag{40}$$

$$\kappa(L_{Tt}(z) + L_{Nt}(z)) = \gamma\alpha_N\frac{\Delta_{Tt}}{P_{Tt}}W_{Nt}(z). \tag{41}$$

Optimization for the firm in the non-tradable sector yields

$$\frac{\alpha_N P_{Nt}(z) y_{Nt}(z)}{L_{Nt}(z)} = W_{Nt}(z), \tag{42}$$

$$\frac{(1 - \alpha_N - \beta_N)P_{Nt}(z)y_{Nt}(z)}{O_{Nt}(z)} = P_{Ot}, \tag{43}$$

$$\beta\frac{\Delta_{Tt}}{P_{Tt}}P_{Nt}(z)\left[\frac{\beta_N y_{Nt}(z)}{K_{Nt}(z)} + (1 - \delta_{KN})\right] = P_{Nt-1}(z)\frac{\Delta_{Tt-1}}{P_{Tt-1}}. \tag{44}$$

The necessary conditions for the representative firm in the tradable sector are

$$\frac{\alpha_T P_{Tt} y_{Tt}}{L_{Tt}} = W_{Tt}, \tag{45}$$

$$(46) \qquad \frac{(1-\alpha_T-\beta_T)P_{Tt}y_{Tt}}{O_{Tt}} = P_{Ot},$$

$$(47) \qquad \beta\Delta_{Tt}\left[\frac{\beta_T y_{Tt}}{K_{Tt}} + (1-\delta_{KT})\right] = \Delta_{Tt-1}.$$

Finally, optimization for the government implies

$$(48) \qquad \frac{T_{t-1}}{P_{t-1}} = \delta(1+r^*)\frac{T_t}{P_t},$$

$$(49) \qquad \frac{\varkappa}{G_{Tt}} = \eta\frac{P_T t}{P_t}T_t,$$

$$(50) \qquad \frac{1-\varkappa}{G_{Nt}} = \eta\frac{P_{Nt}}{P_t}T_t.$$

We adopt the usual symmetry assumptions of the type

$$(51) \qquad y_{Nt}(z) = y_{Nt},$$

which allows us to drop the indexation by the household/firm-specific index z. A specificity of our model is that, with the symmetry assumption, equations (17), (19) and (20) in the problem of the aggregating firm in the non-tradable sector reduce to identities. Effectively, this means that the parameter θ ceases to play a role in the model and, consequently, does not appear in equations (37)-(50). However, the presence of the aggregating firm in the non-tradable sector is convenient from a conceptual standpoint and we retain it in the description of section 2.

In addition to the first-order optimality conditions listed above, a complete specification of the model requires the inclusion of the state equations (2), (9), (15), (25) and (32), as well as the market clearing condition (18). We also include certain identities that provide the definitions of important variables, such as (33) and (34).

Our general strategy for producing simulations with the model is standard and involves the computation of a steady state for the model. Once a (stable) steady state has been found, we produce impulse responses for the endogenous variables with respect to the various shocks included in the model.

In computing the steady state we arrive at a typical situation where equation (48) reduces to a relationship between certain model parameters for a nontrivial stationary point of the system. In order to have a steady state we impose the usual interest-time discounting consistency requirement on the relevant parameters:

$$\delta(1 + r^*) = 1. \tag{52}$$

Since the above implies the loss of one equation from the algebraic system for the steady state, we add another relationship that is required to hold in the long-run equilibrium. In this case we fix the ratio of government expenditures to production ν in equation (53) below.

$$(P_T G_T + P_N G_N) = \nu(P_T y_T + P_N y_N) \tag{53}$$

This choice was made due to the fact that government expenditures to GDP is one of the reference indicators traditionally tracked by various governments for the Bulgarian economy. Needless to say, other calibration schemes are equally possible depending on the particular notion of equilibrium and the specificities of the issue under investigation.

Under the above calibration scheme, we sequentially reduce the algebraic system for the steady state until arriving at a subset of equations that cannot be solved analytically. We then solve this reduced system numerically to obtain approximate values of the respective variables in the steady state. The rest of the steady state values are obtained by tracing back through the reduction steps and substituting the already computed quantities. All the details of the computation procedure are relegated to the appendix. The computed approximate steady state solves the original model system with a high degree of accuracy and satisfies the Blanchard-Kahn conditions (Blanchard and Kahn, 1980).

4 Model calibration and solution

In line with the usual practice, in calibrating the model we choose some of the parameter values on theoretical grounds, while others are determined by empirical considerations. The time discount parameters β and δ are typical representatives of the former category. The approach chosen with respect to the latter category is more flexible, with some parameters calibrated to approximate the current state of the economy (e.g. γ, χ), others calibrated to reflect our views on the likely long-run state of the economy (e.g. ν) and a third group chosen to produce reasonable impulse responses in the simulations conducted (e.g. η, λ). Table 1 in the appendix contains the calibrated values of the model parameters we work with.

The calibration approach sketched above constitutes a first attempt to tune the model to the characteristics of the Bulgarian economy and as such is merely illustrative. Alternative calibration choices along the same lines are certainly possible. More sophisticated approaches, such as calibrating the model to formally match the properties of a suitable data set for the Bulgarian economy or estimating some of the model parameters, should be explored and implemented in future work on the model.

To give the reader some intuition as to what our calibration scheme implies, we summarize in Table 2 (see Appendix 1) the key characteristics of the steady state for the model. All variables presented in the table are in nominal terms and are expressed as a share of steady state GDP [3]. All aggregate variables are defined, where applicable, as the sum of the respective components for the tradable and non-tradable sectors (e.g. household consumption is defined as $P_T C_T + P_N C_N$ etc.).

Equipped with the particular steady state, we proceed in a standard manner and study the behaviour of the endogenous variables around that steady state when the model economy is disturbed by various shocks. All model solution procedures and simulations reported in this and the next section were implemented in Dynare[4].

5 Simulations

The present section illustrates the dynamic behaviour entailed by our model under three scenarios: a shock in the prices of energy resources, a shock in the productivity of the non-tradable sector and a shock in tradable goods prices. We present graphically the impulse responses for relevant selections of model variables. For a set of figures presenting a simulation scenario we follow the convention that the first impulse response function shown is always for the variable directly affected by the shock. The subsequent impulse response functions are ordered according to the

[3] We alert the reader to an accounting technicality in the model: due to the presence of purely imported inputs (energy resources), there arises a distinction between production (gross output), which equals $P_T y_T + P_N y_N$ and GDP (value added), which equals gross output less expenditures on imported inputs. Thus, the calibration choice in equation (53) and Table 1 does not carry over directly to Table 2.

[4] See http://www.cepremap.cnrs.fr/juillard/mambo/index.php

economic logic of the scenario at hand. In all figures the results are transformed so that the zero level corresponds to the steady state value for the respective variable.

The results for the first simulation – a shock in the prices of energy resources – are presented in Figures 1-7 in the Appendix. The initial shock on international energy prices, which dies out slowly over time by virtue of the autoregressive specification, is reflected fully in the dynamics of domestic energy prices. Higher energy prices result in lower output both for the tradable and for the non-tradable sector (Figure 1).

In terms of the sources of the output contraction, consumption of the more expensive energy inputs is unambiguously lower in both sectors (Figure 2). The capital stock in the non-tradable sector rises initially to compensate for the reduction in energy consumption and then falls below its steady state level. Both the capital stock (Figure 2) and the amount of labour used (Figure 3) decline in the tradable sector.

As the demand for labour in the tradable sector declines, so do wages in the sector. This spills over to wages in the non-tradable sector, which also decline, ultimately resulting in higher demand for labour in the non-tradable sector (Figure 3). The aggregate result is lower employment as compared to its steady state level and an overall lower wagebill (Figure 4).

The reason for the somewhat asymmetric response of the firms in the tradable vs. those in the non-tradable sector is apparent in Figure 4. Firms in the non-tradable sector can influence prices in the sector in order to adapt to the increase in energy prices, while prices of tradable goods are unchanged by virtue of the PPP condition (4). Thus, producers in the non-tradable sector have additional flexibility in maximizing their profits, which results in more complicated dynamics of their production inputs. In this particular case, there is a compensating increase in non-tradable goods prices, which is also reflected in the overall price level P.

With lower wages and more expensive non-tradable goods, households curtail their consumption, as displayed in Figure 5. Households also increase their money holdings to try to offset the effects of inflation. In the initial periods after the shock arrives there is some increase in the asset stock as postponing consumption becomes more attractive in view of the higher current prices.

The dynamics of the main variables for the government are shown in 6. Government consumption of non-tradable goods declines in response to the price increase, since by specification the government behaves like a typical consumer and has a standard demand function. The dynamics of taxes in real terms can be explained by observing that they mirror the impulse response function for the overall price level. This means that the government, while preferring to have lower taxes (recall equation (29)), is inclined to behave opportunistically and raise taxes at times when the monetary benefit from doing so is maximized. Correspondingly, the initial negative impact on the level of the government deposit is corrected by means of running budget surpluses. Overall, the specification of the problem for the government sector implies that, in pursuing its interests, the government behaves procyclically.

Finally, as can be expected in the case of a negative external shock, both foreign reserves and the current account balance decline relative to their steady state levels, ultimately returning to equilibrium in a non-oscillatory manner as the shock dies out. The dynamics of foreign reserves reveals the importance of the currency board mechanism, as captured by equation (30), as the behaviour of foreign reserves is determined by the interaction of two factors working in opposite directions – the increase in nominal money holdings and the decline in the government deposit.

The results for a simulation with a shock on productivity in the non-tradable sector are summarized in Figures 8-14. The increase in productivity makes it possible to increase output in the non-tradable sector, which brings about a decline in the output of the tradable sector (Figure 8).

In terms of the sources of output dynamics (Figures 9 and 10), inputs in the non-tradable sector generally increase, while displaying some fluctuations around their steady state levels. Input use in the tradable sector declines unambiguously. Overall, the demand for labour in the economy is dominated by the behaviour of the tradable sector and declines (Figure 11), except in the initial periods, which is reflected in the dynamics of wages and the aggregate wagebill.

The positive productivity shock allows firms in the non-tradable sector to lower prices, thereby lowering the overall price level (Figure 11). This leads to increased consumption of the cheaper non-tradable goods and to some spillover to consumption of tradable goods (Figure 12). The process is associated with a temporary decrease in asset holdings to finance the increase in consumption, followed by a compensating increase before returning to the steady state. Money dynamics mirror those of assets.

The behaviour of the government is again procyclical (Figure 13). Government purchases of non-tradable goods increase in real terms (government consumption of tradable goods is unchanged as their prices remain constant) and taxes are lowered according to the change in the price level. The aggregate outcome for the government is an initial increase in the level of the government deposit, which is corrected by temporarily running budget deficits.

As a result of the increase in the government deposit, foreign reserves increase, despite the reduction in nominal money balances. In contrast, increased private consumption of tradable goods coupled with lower output in that sector leads to a deterioration of the current account (Figure 14).

Our third scenario (Figures 15-21) shows the response of the economy to a shock in the prices of tradable goods, which feeds through the PPP condition (4). Predictably, output in the tradable sector rises while non-tradable sector output declines initially before going slightly above its steady state level.

The demand for labour in the tradable sector increases, pushing wages up, which drains labour from the non-tradable sector, ultimately increasing wages there as well (Figure 16). The aggregate effect is an increase in employment and a higher nominal wagebill (Figure 17).

The use of capital and energy resource inputs increases in the tradable sector, while the adjustment in the non-tradable sector is more complex (Figures 17 and 18). First, firms in the non-tradable sector try to substitute away from the more expensive labour input and therefore raise their consumption of energy resources. As new capital is installed by sacrificing some output that could otherwise be sold on the market, firms in the non-tradable sector initially prefer to forgo some investment (thus lowering their capital stock) in favour of supplying more goods on the market, while increasing their prices. This strategy for the initial periods allows them to have higher revenues with somewhat lower capital stock, which later is compensated by increasing the capital stock above its steady state level. As a result of higher prices of both tradable and non-tradable goods, the overall price level goes up.

Against the background of the favorable income developments, households increase their consumption (Figure 19). At the same time, in order to compensate for the higher prices, households increase their nominal money holdings . These, as well as higher taxes levied by the government (see below), lead to a drop in the asset stock in the initial periods.

The government sector reduces its consumption of both tradable and non-tradable goods in response to the price increases (Figure 20). At the same time, the higher aggregate price level induces the government to levy higher real taxes, which boosts nominal tax revenues. The overall outcome for the government is a reduction in the government deposit (Figure 21), which is adjusted by running budget surpluses.

The reduction in the government deposit with the central bank causes a corresponding reduction in the level of foreign reserves. Higher prices of tradable goods provide sufficient incentive to firms in the sector, so that the increase in their output is enough to cover higher private consumption of tradable goods and higher imports of energy inputs. The balance is exported and a current account surplus results (Figure 21).

6 Conclusions

The responses of the model variables with respect to various shocks that affect the economy are plausible and correspond to usual consequences entailed by the main economic mechanisms built into the model. In general, shocks that are traditionally considered "positive" translate into economic expansions in the present framework. Conversely, "negative" shocks lead the model economy into recession. Government behaviour also affects the economy in an intuitive way, with the caveat that government priorities do not take into account the impact of its actions on other economic agents. Overall, the above implies that the present work can serve as a platform for building more sophisticated and realistic dynamic models of the Bulgarian economy.

The results obtained in the present work suggest a number of directions for extending and improving the model. The most urgent need for improvement concerns the specification of the government sector. The current preference structure for the government implies behaviour that is little concerned with issues of macroeconomic stability or social welfare. In order to have a setup featuring an active government, which at the same time acts responsibly in the interests of society, the current specification has to be modified to take into account broader macroeconomic concerns. This in turn will probably necessitate the introduction of a more detailed set of policy instruments at the government's disposal, including a set of various tax rates.

A second direction for modifying the model would be the introduction of nominal rigidities and adjustment costs in the analytical framework. These could include both price and wage rigidities, which presumably would lead to more realistic dynamics in the model. Additionally, the possibility of introducing adjustment costs for capital/investment should also be investigated. Depending on the specification details, this could serve the standard purpose of moderating excessive fluctuations in investment or could be used to model more complex phenomena that arise, for instance, in connection with the decisions of foreign direct investors.

One more technical level, a useful modification of the implementation framework would be to allow simulations starting from predefined initial conditions and reaching a particular steady state, if feasible. This depends on the model characteristics (saddle path property of the system of necessary conditions), as well as the availability of a suitable platform for implementation. In the latter respect, the Anderson-Moore (AIM) algorithm (Anderson, 2000) and the software tools implementing it[5] seem to be a promising direction to explore.

[5] See `http://www.federalreserve.gov/pubs/oss/oss4/aimindex.html` for an extensive list of references and codes related to the AIM algorithm.

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Appendices

Appendix 1 – figures and tables

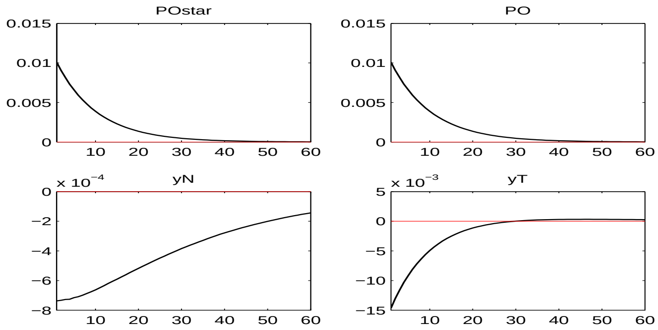

Figure 1: Impulse responses to a shock in energy prices for P_O^*, P_O, y_N and y_T.

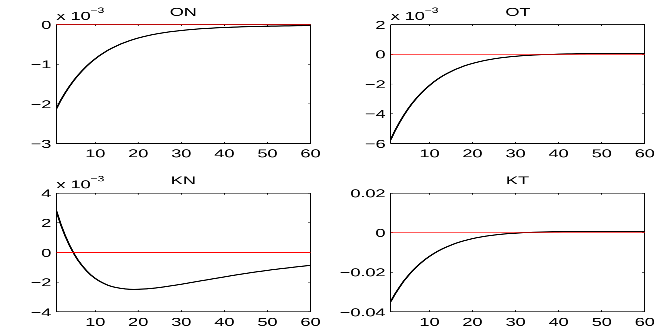

Figure 2: Impulse responses to a shock in energy prices for O_N, O_T, K_N and K_T.

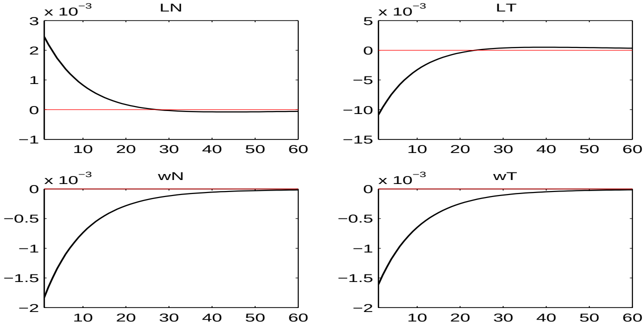

Figure 3: Impulse responses to a shock in energy prices for L_N, L_T, W_N and W_T.

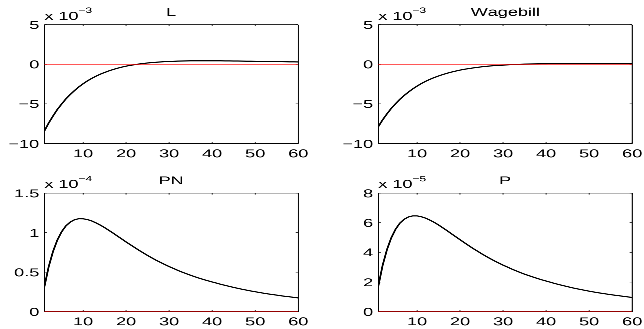

Figure 4: Impulse responses to a shock in energy prices for L, WB, P_N and P.

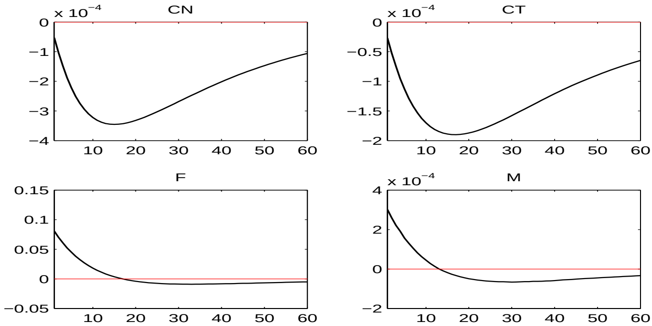

Figure 5: Impulse responses to a shock in energy prices for C_N, C_T, F and M.

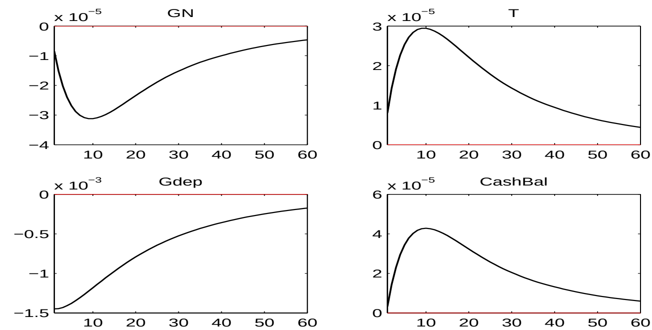

Figure 6: Impulse responses to a shock in energy prices for G_N, T, Γ and CB.

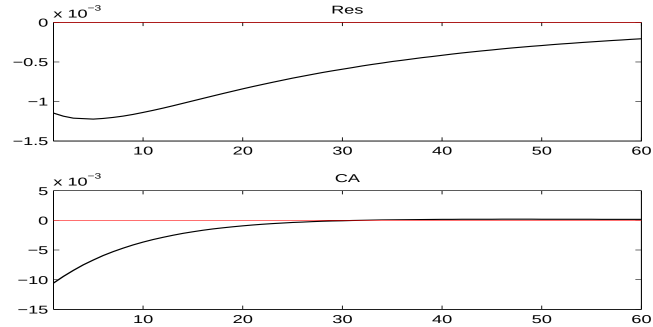

Figure 7: Impulse responses to a shock in energy prices for $P_T F^*$ and CA.

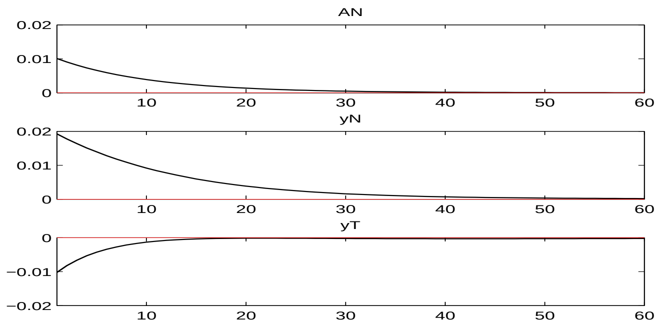

Figure 8: Impulse responses to a shock in non-tradable sector productivity for A_N, y_N and y_T.

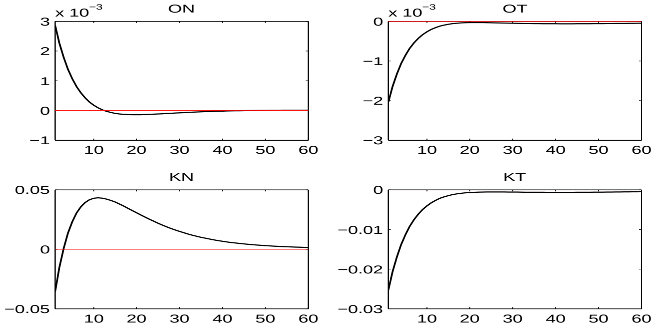

Figure 9: Impulse responses to a shock in non-tradable sector productivity for O_N, O_T, K_N and K_T.

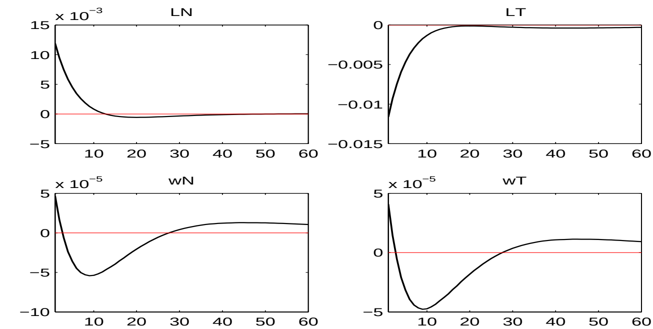

Figure 10: Impulse responses to a shock in non-tradable sector productivity for L_N, L_T, W_N and W_T.

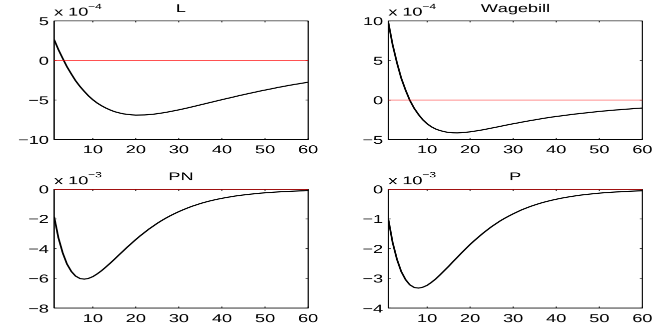

Figure 11: Impulse responses to a shock in non-tradable sector productivity for L, WB, P_N and P.

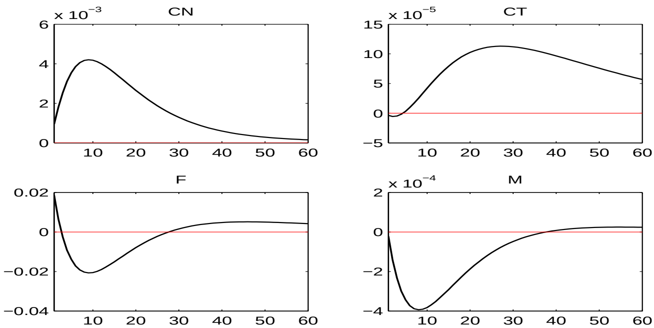

Figure 12: Impulse responses to a shock in non-tradable sector productivity for C_N, C_T, F and M.

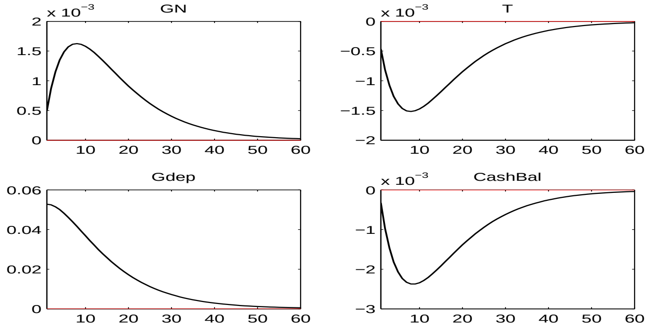

Figure 13: Impulse responses to a shock in non-tradable sector productivity for G_N, T, Γ and CB.

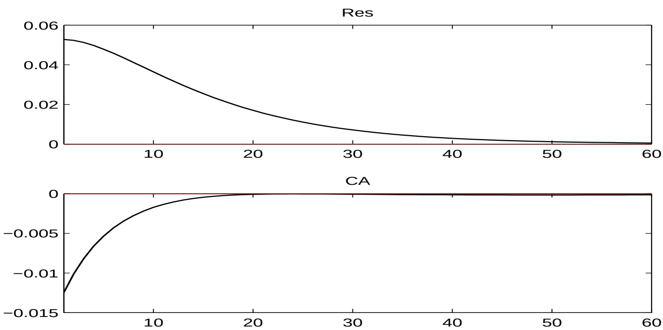

Figure 14: Impulse responses to a shock in non-tradable sector productivity for $P_T F^*$ and CA.

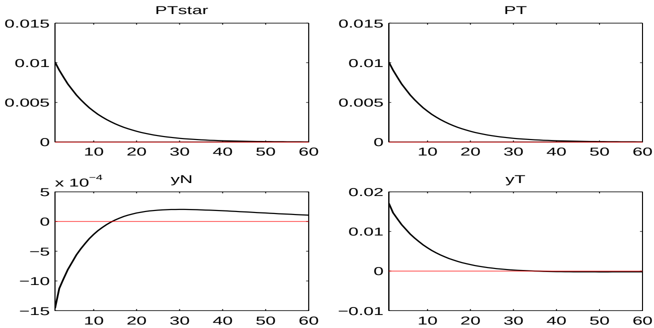

Figure 15: Impulse responses to a shock in tradable goods prices for P_T^*, P_T, y_N and y_T.

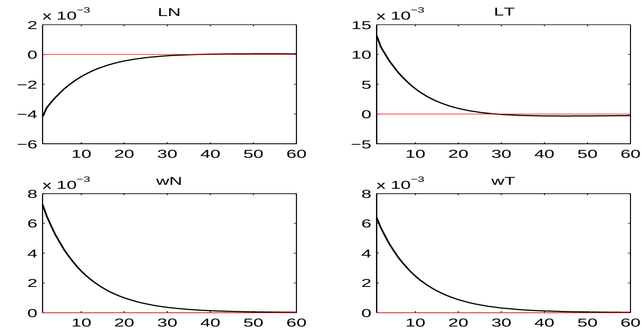

Figure 16: Impulse responses to a shock in tradable goods prices for L_N, L_T, W_N and W_T.

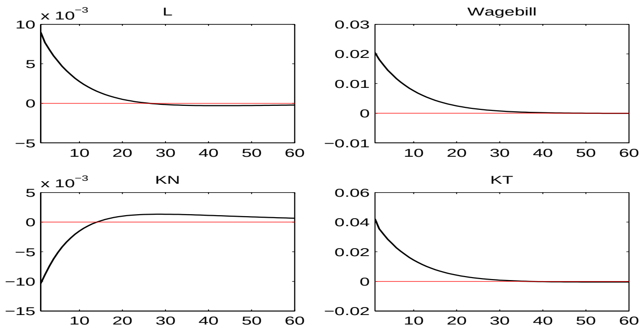

Figure 17: Impulse responses to a shock in tradable goods prices for L, WB, K_N and K_T.

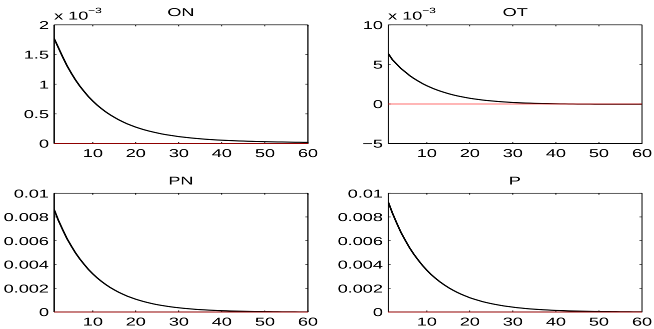

Figure 18: Impulse responses to a shock in tradable goods prices for O_N, O_T, P_N and P.

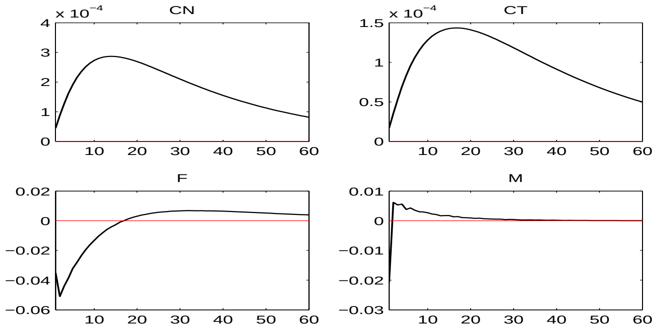

Figure 19: Impulse responses to a shock in tradable goods prices for C_N, C_T, F and M.

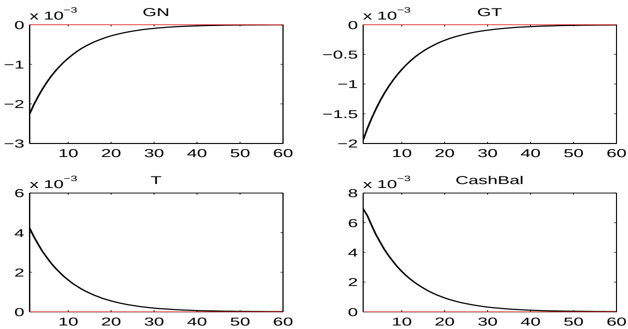

Figure 20: Impulse responses to a shock in tradable goods prices for G_N, G_T, T and CB.

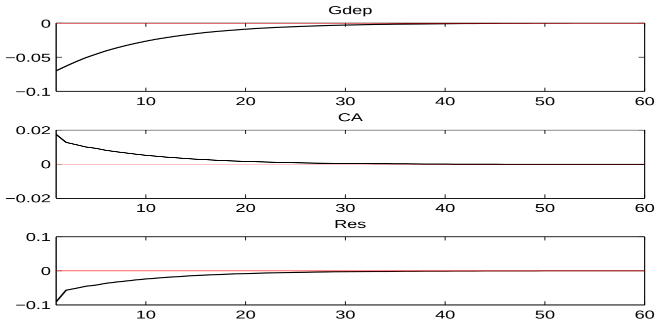

Figure 21: Impulse responses to a shock in tradable goods prices for Γ, CA and $P_T F^*$.

Para-meter	Interpretation	Value
α_T	Production function coefficient (labour, tradable sector)	0.52
α_N	Production function coefficient (labour, non-tradable sector)	0.44
β	Intertemporal discount factor (household)	0.98
β_T	Production function coefficient (capital, tradable sector)	0.28
β_N	Production function coefficient (capital, non-tradable sector)	0.36
b_T	Habit formation parameter (tradable goods consumption)	0.5
b_N	Habit formation parameter (non-tradable goods consumption)	0.6
χ	Real money balances preference parameter	0.0007
δ	Intertemporal discount factor (government)	0.99
δ_{KT}	Depreciation (tradable sector)	0.09
δ_{KN}	Depreciation (non-tradable sector)	0.07
ϵ	Relative risk aversion coefficient (real money)	2
η	Disutility of taxes parameter	5.6
γ	Tradable goods preference parameter (household)	0.45
κ	Disutility of labour parameter	0.1
λ	Labour aggregation technology coefficient	2
l^*_{PO}	Steady-state level of energy prices	1
l^*_{PT}	Steady-state level of tradable goods prices	1
ν	Ratio of government expenditures to production	0.16
ψ	Risk premium weight	0.0005
ρ_E	Autoregressive coefficient (exchange rate)	0.9
$\bar{F}$	Perceived equilibrium asset position	0
ρ_T	Autoregressive coefficient (tradable sector productivity)	0.9
ρ_N	Autoregressive coefficient (non-tradable sector productivity)	0.9
ρ_{PO}	Autoregressive coefficient (energy prices)	0.9
ρ_{PT}	Autoregressive coefficient (tradable goods prices)	0.9
$\varkappa$	Tradable goods preference parameter (government)	0.5
i^*	International interest rate	$1/\beta - 1$
r^*	Interest rate on foreign reserves	$1/\delta - 1$
E_L	Steady-state level of exchange rate	1

Table 1: Calibrated values of model parameters

Variable	Value
Household consumption	49.1%
Investment	31.1%
Government expenditures	20.0%
Capital	391.4%
Wagebill	60.9%
Profit	8.0%
Government deposit	11.9%
Money	8.8%
Tax revenues	19.8%
Current account	0.0%
Budget balance	0.0%

Table 2: Key characteristics of the steady state (in % of GDP)

Appendix 2 – notes on the solution of the system of equations for the steady state

The notations for the steady-state values of the different variables are the same as in section 3, with the obvious omission of the time indices t. Define

$$B_N := \frac{1-\beta b_N}{1-b_N}, \quad B_T := \frac{1-\beta b_T}{1-b_T}.$$

We obtain

$$\Delta_N = \frac{B_N}{C_N}, \quad \Delta_T = \frac{B_T}{C_T}.$$

The system of equations for the steady state, derived on the basis of the dynamic equations in section 3 and the respective constraints in the different problems, has the form:

(A.1) $$\frac{(1-\gamma)B_N}{C_N P_N} = \frac{\gamma B_T}{C_T P_T},$$

(A.2) $$\beta\left(1 + i^* \psi\left[e^{\tilde{F}-F} - 1 - F e^{\tilde{F}-F}\right]\right) = 1,$$

(A.3) $$\left(\frac{M}{P}\right)^{\varepsilon} = \frac{\beta\chi}{(1-\beta)\gamma B_T P} P_T C_T,$$

(A.4) $$\kappa\lambda(L_T + L_N) = \gamma(\lambda - 1)\frac{B_T}{C_T P_T} W_T,$$

(A.5) $$\kappa(L_T + L_N) = \gamma\alpha_N \frac{B_T}{C_T P_T} W_N,$$

(A.6) $$\bar{i} F P_T + W_T L_T + W_N L_N - P_T C_T - P_N C_N + \Pi_N + P i_T - PT = 0,$$

where $\bar{i} = i^* + \psi\left[e^{\tilde{F}-F} - 1\right]$,

(A.7) $$Z = 0,$$

$$\alpha_N A_N P_N L_N^{\alpha_N - 1} K_N^{\beta_N} O_N^{1-\alpha_N-\beta_N} = W_N, \tag{A.8}$$

or, equivalently,

$$\alpha_N P_N y_N = W_N L_N, \tag{A.8$'$}$$

$$(1-\alpha_N-\beta_N) A_N P_N L_N^{\alpha_N} K_N^{\beta_N} O_N^{-\alpha_N-\beta_N} = P_O, \tag{A.9}$$

or, equivalently,

$$(1-\alpha_N-\beta_N) P_N y_N = P_O O_N, \tag{A.9$'$}$$

$$\beta \left[\beta_N A_N L_N^{\alpha_N} K_N^{\beta_N - 1} O_N^{1-\alpha_N-\beta_N} + (1-\delta_{KN})\right] = 1, \tag{A.10}$$

or, equivalently,

$$\beta_N y_N = (\beta^{-1} - 1 + \delta_{KN}) K_N, \tag{A.10$'$}$$

$$I_N = \delta_{KN} K_N, \tag{A.11}$$

$$\alpha_T A_T P_T L_T^{\alpha_T - 1} K_T^{\beta_T} O_T^{1-\alpha_T-\beta_T} = W_T, \tag{A.12}$$

or, equivalently,

$$\alpha_T P_T y_T = W_T L_T, \tag{A.12$'$}$$

$$(1-\alpha_T-\beta_T) A_T P_T L_T^{\alpha_T} K_T^{\beta_T} O_T^{-\alpha_T-\beta_T} = P_O, \tag{A.13}$$

or, equivalently,

$$(1-\alpha_T-\beta_T)P_T y_T = P_O O_T, \tag{A.13$'$}$$

$$\beta\left[\beta_T A_T L_T^{\alpha_T} K_T^{\beta_T-1} O_T^{1-\alpha_T-\beta_T} + (1-\delta_{KT})\right] = 1, \tag{A.14}$$

or, equivalently,

$$\beta_T y_T = (\beta^{-1} - 1 + \delta_{KT})K_T, \tag{A.14$'$}$$

$$I_T = \delta_{KT} K_T, \tag{A.15}$$

$$\delta(1+r^*) = 1, \tag{A.16}$$

$$\eta\frac{T}{P} = \frac{\varkappa}{P_T G_T}, \tag{A.17}$$

$$\eta\frac{T}{P} = \frac{1-\varkappa}{P_N G_N}, \tag{A.18}$$

$$r^*(\Gamma + M) + PT - P_T G_T - P_N G_N = 0, \tag{A.19}$$

$$I_N + C_N + G_N = y_N. \tag{A.20}$$

We start the process of solving the system by numerically computing the value of F from equation (A.2). From this point on there are several ways of reducing the system algebraically. The approach we describe below was chosen on the basis of the fact that it does not lead to expressions featuring ratios of small numbers. We found that in alternative reduction schemes, where fractions involving small numbers are present, the precision of the numerical computations is greatly reduced.

To implement the present reduction scheme, we divide (A.4) by (A.5) to obtain

$$W_N = \frac{\lambda - 1}{\lambda \alpha_N} W_T \tag{A.21}$$

(cf. Natalucci and Ravenna (2002), p.9, equations (21), (22) and their implications).

Let us raise (A.12′), (A.13′) and (A.14′) to the power α_T, $1-\alpha_T-\beta_T$ and β_T, respectively, and multiply them. We get

$$\begin{aligned} W_T = \Bigg\{ & A_T(\alpha_T P_T)^{\alpha_T} \left[\frac{(1-\alpha_T-\beta_T)P_T}{P_O}\right]^{1-\alpha_T-\beta_T} \times \\ & \times \left[\frac{\beta_T}{\beta^{-1}-1+\delta_{KT}}\right]^{\beta_T} \Bigg\}^{\frac{1}{\alpha_T}}. \end{aligned} \tag{A.22}$$

Now (A.21) yields a formula for W_N.

Similarly, multiplying (A.8′), (A.9′) and (A.10′), raised to the power α_N, $1-\alpha_N-\beta_N$ and β_N, respectively, we obtain a formula for P_N:

$$\begin{aligned} P_N^{1-\beta_N} = & \frac{1}{A_N}\left(\frac{W_N}{\alpha_N}\right)^{\alpha_N}\left(\frac{P_O}{1-\alpha_N-\beta_N}\right)^{1-\alpha_N-\beta_N} \times \\ & \times \left(\frac{\beta^{-1}-1+\delta_{KN}}{\beta_N}\right)^{\beta_N}. \end{aligned} \tag{A.23}$$

Equations (A.20), (A.11) and (A.10′) yield

$$C_N + G_N = (1-\mu_N)y_N, \tag{A.24}$$

where $\mu_N = \beta_N \delta_{KN}/(\beta^{-1}-1+\delta_{KN})$.

Let us write condition (53) in the form

$$P_N y_N + P_T y_T = \frac{P_N G_N}{(1-\varkappa)\nu}. \tag{A.25}$$

Since we have

$$\begin{aligned} \Pi_N + W_N L_N &= P_N y_N(\alpha_N + \beta_N - \mu_N), \\ \Pi_T + W_T L_T &= P_T y_T(\alpha_T + \beta_T - \mu_T), \end{aligned}$$

where $\mu_T = \beta_T \delta_{KT}/(\beta^{-1} - 1 + \delta_{KT})$, equation (A.6) is transformed to

(A.26) $\bar{i}FP_T + P_N y_N(\alpha_N + \beta_N - \mu_N) + P_T y_T(\alpha_T + \beta_T - \mu_T) = VC_N + PT,$

where

$$V = \left(1 + \frac{\gamma B_T}{(1-\gamma)B_N}\right) P_N.$$

Next, rewrite equation (A.18) as

$$TG_N = W,$$

where

$$W = \frac{(1-\varkappa)P}{\eta P_N}.$$

Finally, taking into account (A.8′) and (A.12′), which provide expressions for L_N and L_T, equation (A.5) becomes

$$\frac{\lambda \alpha_N^2}{\lambda - 1} P_N y_N + \alpha_T P_T y_T = \frac{W_T}{C_N} U, \tag{A.27}$$

where

$$U = \frac{\alpha_N (1 - \gamma B_N W_N)}{\kappa P_N}.$$

If we denote

$$P_N y_N = n, \quad P_T y_T = t,$$

the system (A.24), (A.25), (A.26) and (A.27) in the unknowns n, t, C_N, G_N can be written as

$$\begin{cases} n = \frac{P_N}{1-\mu_N}(C_N + G_N), \\ n + t = \frac{P_N}{(1-\varkappa)\nu} G_N, \\ \bar{i}FP_T + (\alpha_N + \beta_N - \mu_N)n + (\alpha_T + \beta_T - \mu_T)t = \\ \qquad = VC_N + \frac{PW}{G_N}, \\ \frac{\lambda\alpha_N^2}{\lambda-1} n + \alpha_T t = \frac{W_T}{C_N} U. \end{cases} \tag{A.28}$$

We can sequentially express n, t and C_N from the first three equations of the system (A.28) and substitute them in the last equation. Then we numerically solve for G_N from this last equation and use it to substitute back in the previous formulae and recover the rest of the steady-state values. All numerical computations were carried out using the built-in solvers available in MATLAB®.

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