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SUMMARY. We propose the use of penalized splines as a new methodology for the estimation of potential output.

We begin by presenting two frequently used approaches for the estimation of potential output, namely the HP filter on hand and the normal linear state-space approach on the other hand. We compare these two methods to the penalized spline approach on Bulgarian data.

We argue for the advantages of penalized splines routine use over the approaches commonly used in central banks.

Key words: potential output, output gap, scatterplot smoothing, state-space models, mixed models.

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1 Introduction

Estimation of the trend in the GDP series is an important and useful exercise. The output series is supposed to have two components: the trend component (which is also called the potential output component), and the cyclical component or noise (which is also called the output gap component).

Usually, this whole task is undertaken under the name of "potential output" estimation, but we prefer here to think of it more as a procedure to separate the trend from the noise in the data.

It is carried out in almost all central banks all over the world and it is used not only as an informative data exercise but sometimes as a policy tool. For instance, it can be used to determine interest rates set by the monetary authorities in a Taylor rule framework. Also, the output gap variable can be used instead of unemployment data in a Phillips curve-like estimation exercise.

There are two classes of methods for determining potential output and these are: the structural methods and the purely statistical ones. Structural approaches use some economic model (such as aggregate production functions models) to determine the potential level of output. The purely statistical methods do not rely on any economic model and are only concerned with the statistical characteristics of the trend (or noise) component and use those statistical characteristics to find a way to identify the different components.

The most widely used method for potential output estimation is the Hodrick Prescott filter, and it is a purely statistical approach. It is used in most central banks because of its simplicity and because of the possibility of applying it on a routine basis in a conformable way that would not give rise to misunderstandings across a wide range of institutions (such as, for instance, the different central banks in the European union). We are going to quickly present this method in a subsequent section and argue against its usage.

Next to the Hodrick-Prescott filter, there are a wide range of statistical procedures that can all be put in one big category, which is the linear and normal state-space models. Methods in this category come under a wide range of different names (such as polynomial trend component models, unobserved components models, latent variables models.) and sometimes they are referred to under the name of the Kalman filter (which is a misnomer, since the Kalman filter is nothing but the common way of estimating the latent variables in such models given the other parameters therein. We are going to go into more detail regarding this point in a latter section).

State-space models are quite versatile and do not suffer from the usual critiques directed against the HP filter. (Nonetheless, in a certain sense, the HP filter can be studied in the state-space framework!) Therefore, why propose yet another estimation procedure? The reason behind it is that, due to the presence of several alternatives for parameterizing the different models, and due to the (relative) complexity of state-space models, the end user is usually perplexed and does not know which procedure to pick up. Testing and selecting between different state-space models might prove to be a gargantuan task for recovering and estimating something as simple as the potential output.

It is mainly because of this that we propose to use penalized splines. Penalized splines are widely used in semiparametric statistics. There are used to fit nonlinear regression models and generalized additive models [Hastie and Tibshirani, 1996]. There are used in a variety of fields such as geostatistics, biostatistics and more recently econometrics. The paper by [Eilers and Marx, 1996] contributed to making this type of models very popular. [Ruppert et al., 2003] gives a wide range of applications and it also explains the main aspects of the methodology in a very simple and intuitive framework.

Beside their simplicity and the fact that they do not suffer from the main criticisms addressed to the HP filter methodology, they present two more advantages. First, they can be used on a routine basis and everything can be selected automatically in the software, which means that the end-user does not need to go into too many details for fitting and testing the models. Moreover, it can be considered in the general framework of state-space models (but we are not going to explore much further that point). Besides those two points, there are many more attractive features. We are going to explore all the advantages in a later subsection.

We also are going to explore all the previous points of the introduction in much detail. Here is an outline of the rest of the paper. We begin by presenting the standard HP approach in section 2. We use this as a benchmark for later comparison proposes. We also present one of the normal linear state-space approaches in that section. We introduce our main approach (the penalized splines approach) in section 3. In that section we also present some empirical results relating to Bulgaria's GDP (For previous work on potential output estimation, see [Tsalinski, 2005] and the references therein). In section 4 we compare the three main approaches undertaken in this paper on Bulgarian data. Chapter 5 concludes. There is also an appendix on a structural nonlinear and nonnormal state-space approach that was also considered by the author.

2 Preliminaries

We present two main approaches to potential output estimation in this section. These are the Hodrick-Prescott approach and one of the linear and normal state-space approaches.

2.1 The HP filter

As already discussed, the HP filter is the most widely used approach for potential output estimation. We are going to explain how it works, give some rationalizations behind its use and then, we will present its main disadvantages, which would allow us to argue against its use.

To fix notation, we recall that the objective of potential output estimation is to decompose the output series y_t into two components

- a trend component (or potential component) τ_t.
- and a cyclical component (or output gap) c_t.

Therefore, the y_t series can be written as

$$y_t = \tau_t + c_t$$

The way the HP filter is constructed is through the solution of the following minimization problem

Find the sequence $\{c_t\}_{t=1:T}$ that minimizes

$$\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=3}^{T} \left(\nabla^2 \tau_t^2\right)$$

This can be rationalized in the following way: The objective is to minimize the cyclical (noise) component under the constraint that the second difference of the trend component be equal to zero (i.e. the trend component should be smooth). In this framework, λ is interpreted as the Lagrange multiplier associated with the constraint that the second difference of the trend component be equal to zero.

Another way of looking at it is to interpret the objective function as a loss function where there is a trade-off (tuned by λ) between how large is the noise and how smooth is the trend.

Another (frequency domain) interpretation of the HP filter is to see it as a special case of the Butterworth family of filters. For this and many more materials on the HP filter, look at the excellent reference of [Kaiser and Maravall, 2001].

The way the filter is implemented is by solving the minimization problem analytically through the use of a linear operator.

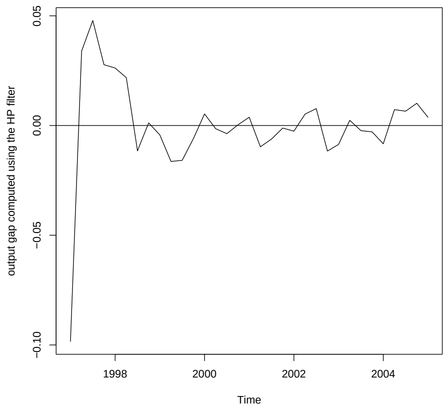

Figure 1. Bulgarian output gap computed using the HP filter.

Pros and Cons

The main advantage of the HP filter is that, as mentioned earlier for several times, it is extremely simple and that it can be implemented on a routine basis.

However, the HP filter suffers from the following (widely accepted) criticisms

1. It is weak at the endpoints of the sample path. Note that, in order to get the second difference of the trend component, we lose some information. The minimization problem is solved using the linear operator in the middle of the sample but not at the endpoints. This is extremely severe because the last observations are usually considered the most important ones. This problem would cause the monetary authorities to make frequent revisions and sometimes, the magnitude of those revisions might be important.

2. The choice of λ is ad-hoc. For quarterly data, it is accepted to be equal to 1600. Note that the λ parameter is very important. Choosing too high a value for λ would over-smooth the fit. Choosing too low a value would overfit the data.

3. The use of the HP filter could induce some spurious results (spurious cross-correlations, spurious autocorrelations or spurious periodic cycles).

4. The HP filter approach is not a model-based one. It is not anchored into a rigorous statistical model and therefore, it can not be used for other purposes than separating the potential output from the output gap. In other words, it can not be used for forecasting, undertaking statistical inference such as hypothesis testing or constructing confidence intervals unless it is significantly augmented.

The shortcomings of the HP filter are explored in much detail in the work of [Kaiser and Maravall, 2001].

2.2 State-space models

A state-space model consists of a vector of observed variables (say $\boldsymbol{Y}_t$) and a vector of latent (state or unobserved) variables (say $\boldsymbol{\theta}_t$) and it is characterized by two equations.

The first equation is called the measurement equation (or observation equation) and it describes the relation between the observed variables and the state variables.

$$\boldsymbol{Y}_t = f(\boldsymbol{\theta}_t, \boldsymbol{v}_t)$$

where $\boldsymbol{v}_t$ is an error vector of the same dimension of the observed variable. $f(.,.)$ is some appropriate function.

The second equation is called the transition equation (or law of motion equation) and describes the way the state variable evolves over time

$$\boldsymbol{\theta}_t = g(\boldsymbol{\theta}_{t-1}, \boldsymbol{w}_t)$$

where $\boldsymbol{w}_t$ is an error vector of the same dimension as the latent variable. $g(.,.)$ is some appropriate function.

An important special case is the linear normal model

$$\boldsymbol{Y}_t = \boldsymbol{F}_t'.\boldsymbol{\theta}_t + \boldsymbol{v}_t$$

$$\boldsymbol{\theta}_t = \boldsymbol{G}_t.\boldsymbol{\theta}_{t-1} + \boldsymbol{w}_t$$

where $\boldsymbol{F}_t$ and $\boldsymbol{G}_t$ are conformable matrices. $\boldsymbol{v}_t$ and $\boldsymbol{w}_t$ are multivariate normal errors.

For important references on normal linear state-space models see, for instance, [Harvey, 1989] and [West and Harrison, 1997]. For references on the nonlinear and nonnormal models, see [Tanizaki, 2003] and [Pitt and Shephard, 1999].

Some important terminology needs to be introduced in order to be able to describe easily different notions of the normal linear state-space model.

Let $\mathcal{D}_t \equiv \{\boldsymbol{Y}_t, \mathcal{D}_{t-1}\}$ denote the information set (in other words, the history of $\boldsymbol{Y}_t$). The parameters of the model ($\boldsymbol{F}_t'$, $\boldsymbol{G}_t$ and the covariance matrices of $\boldsymbol{v}_t$ and $\boldsymbol{w}_t$) are called the hyperparameters.

The unknowns of the model are the set of hyperparameters and the history of the latent (state) variables $\boldsymbol{\theta}_t$. For the estimation of $\boldsymbol{\theta}_t$, it is important to define the following three different types of densities (and the "action" associated with each density).

$p(\boldsymbol{\theta}_{t+s}\vert\mathcal{D}_t)$	prediction	
$p(\boldsymbol{\theta}_t\vert\mathcal{D}_t)$	filtering	(on-line analysis)
$p(\boldsymbol{\theta}_t\vert\mathcal{D}_{t+s})$	smoothing	(retrospective analysis)

The density $p(\boldsymbol{\theta}_t|\mathcal{D}_t)$ is of particular importance. It is called the density of filtered probabilities. In the linear and normal state-space model, the algorithm that recovers the mean and variance of that density at each point in time given the values of the hyperparameters is called the Kalman filter.

Now, we are going to use the notions introduced above and present one instance of a linear state-space model utilized to recover estimates of potential output. We are going to introduce the important class of polynomial trend models.

2.2.1 The second order polynomial trend model

A way to estimate potential output is to use the second order polynomial trend model. This model is also sometimes referred to as the second order random walk model.

In this model, the output data are supposed to be generated by a random walk model with drift. The drift is a very necessary component since it reproduces the trending up behavior in the data. If the drift is constant, we get the first order polynomial trend model. To allow much more flexibility, we can assume that the drift itself is time-varying. This can be achieved, say, through adding an equation that describes the drift as a random walk itself.

We can write the model in the following way. Output is supposed to have the trend component (potential output) α_t and the noise component (output gap) ε_t.

$$y_t = \alpha_t + \varepsilon_t$$

ε_t is supposed to be normally distributed

$$\varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$$

The trend component is a random walk with time-varying drift β_t.

$$\alpha_t = \beta_t + \alpha_{t-1} + w_{1,t}$$

$$\beta_t = \beta_{t-1} + w_{2,t}$$

$$\boldsymbol{w}_t = \begin{pmatrix} w_{1,t} \\ w_{2,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{w,1}^2 & 0 \\ 0 & \sigma_{w,2}^2 \end{pmatrix}\right)$$

We can write the model into the state-space form by specifying the two equations, which are the measurement equation

$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_t \\ \beta_{t+1} \end{pmatrix} + \varepsilon_t$$

and the state equation

$$\begin{pmatrix} \alpha_t \\ \beta_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_{t-1} \\ \beta_t \end{pmatrix} + \begin{pmatrix} w_{1,t} \\ w_{2,t+1} \end{pmatrix}$$

The vector of latent variables is therefore $\boldsymbol{\theta}_t = \begin{pmatrix} \alpha_t \\ \beta_{t+1} \end{pmatrix}$.

If we know the values taken by the variances σ_ε^2, $\sigma_{w,1}^2$ and $\sigma_{w,2}^2$ (which, as already explained, they are called hyperparameters in the jargon of state-space models), then estimating values assumed by the latent state vector $\begin{pmatrix} \alpha_t \\ \beta_{t+1} \end{pmatrix}$ is quite easy. It can be done using the Kalman filter (whose recursion basically computes the parameters of the density of filtered probabilities, i.e. $p(\boldsymbol{\theta}_t | \mathcal{D}_t)$).

Using the recursions of the Kalman filter, we can compute the likelihood (of the measurement density) which we can maximize in order to obtain the variances σ_ε^2, $\sigma_{w,1}^2$ and $\sigma_{w,2}^2$. Or, from a Bayesian perspective, we can sample from the posterior of the model using a Gibbs sampling procedure consisting basically of two main steps

- Given the hyperparameters, run the Kalman filter to get an estimate of the state variables.
- Given the state variables, perform a set of two linear regression to recover the hyperparameters.

2.2.2 Empirical results

The model was estimated in a Bayesian framework using `WinBUGS` and `R`. The author also wrote some Kalman filtering code in `scilab`.

An initial burn-in period of 5000 iteration was undertaken and a 100000 iteration after burn-in were carried out.

The estimated hyperparameters assume the following values

	mean	standard deviation
σ_ε^2	0.06171067	0.01075058
$\sigma_{w,1}^2$	0.07507902	0.01532877
$\sigma_{w,2}^2$	0.1166716	0.02197357

Given the mean values of the hyperparameters, the Kalman filter was run. The mean values of α_t from the Kalman filter output can be considered as potential output. The difference between y_t and α_t constitutes output gap.

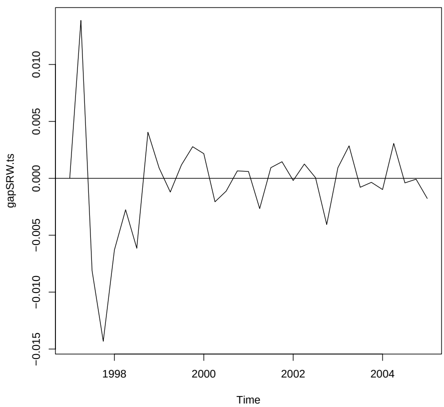

Figure 2. Output gap estimated using the second order trend polynomial model.

2.2.3 Pros and Cons of the linear normal state-space approach

Pros

Linear normal state-space models have many advantageous characteristics. First, they are anchored in a model-based approach in the sense that computing potential output with these techniques is not just a simple ad-hoc algorithm, but one has to specify a complete statistic model for the underlying stochastic model. They are also very flexible regarding statistical inference in the sense that one can easily undertake forecasting tasks or construct confidence bands around the estimates.

A single advantage in comparison with the HP filter is that they are not known to represent the spurious results contaminating the HP filter results and, furthermore, they do not suffer from weakness at the endpoints.

Those are only a few advantages that we mention here. One can think easily of many more such advantages.

Cons

The first problem that we can think of is that the methods based on state-space models like the one exposed earlier are not nonparametric. The have a parametric representation of the data and try to fit that parametric representation. This is ideal if we are sure that the considered parametric model is the process that generated the dataset. However, in most circumstances, we are not sure of that point. Therefore, the model may give rise to some misspecification issues. Moreover, it attempts at giving both the trend component and the noise component a form that is as close as possible to the parametric form posited. In the preceding example, it attempts to give the trend component the form of a random walk with drift. Due to this, we might overfit the data. Suppose that the data were generated by a simple linear trend and some noise, using the trend component model, we might get a trend that is less smooth than the one that generated the data!

Another issue is to be able to determine which model to pick. This might prove to be an overwhelming task. Model selection might be extremely difficult in frequentist inference because we might be comparing non-nested models. In Bayesian analysis, model selection requires the estimation of all the different state-space models using Monte Carlo Markov chains in order to recover the Bayes factors comparing different models. Bayesian model averaging also requires the estimation of all models. Although the state-space approach in this case is extremely appealing, the task of model selection might prove to be Herculean for the central banker. For a topic as output gap estimation, to quote Sir Ronald Fisher, it is like shooting at a bird with a canon and (possibly, in case of misspecification) missing the bird!

3 Penalized splines

3.1 Introduction

Penalized splines are just one of many semiparametric regression procedures. The main idea behind using them for the estimation of potential output is to assume that the trend is some nonlinear function of time (without predetermining any given form for that function) and try to fit that nonlinear model to the data.

The main idea is based on scatterplot smoothing. That is, plot the scatterplot of the data against time, and try to figure a way to draw some smooth function in the scatterplot that would summarize the relation between data and time. The idea is similar to basic justifications behind linear models in which one attempts to fit a line through the scatterplot.

Splines are a generalization of the idea that, to estimate a nonlinear function while still using linear statistical methods, we consider as regressors different powers (i.e a polynomial function in the regressors) of the covariates. This can be thought of as a construction of a basis (of functions) that can span any type of smooth functions that one can think of through the scatterplot. Splines come in handy because they can be thought of as some sort of construction of such a basis using given some points at the covariates called knots.

Let's say we have K knots at the following points κ_k, $k=1,\ldots,K$. Using those knots we can construct the basis functions

$$\begin{pmatrix} 1 & t & (t-\kappa_1)_+ & \cdots & (t-\kappa_K)_+ \end{pmatrix}$$

For the purpose of this paper, there is a single covariate, that is time t.

We also use the definition of the quantity $(t-\kappa_k)_+$

$$(t-\kappa_k)_+ = \begin{cases} t-\kappa_k & \text{if } (t-\kappa_k) > 0 \\ 0 & \text{otherwise} \end{cases}$$

This is called a truncated polynomial basis. We can think also of other basis such as the low-rank thin-plate spline

$$\left(1 \quad t \quad |t-\kappa_1|^3 \quad \cdots \quad |t-\kappa_K|^3 \right)$$

These basis functions can generate all types of smooth (and non-smooth) non-linear forms by taking different linear combinations.

An excellent reference on that topic is [Ruppert et al., 2003].

Now that we have introduced the basic concepts of splines and knots, we can introduce our general framework for modeling potential output.

3.2 The procedure

As previously explained, we would like to allow the shape of the trend component to take any nonlinear smooth form and we would like the data to give us such form. We can think of the linear model

$$y_t = f(t) + \varepsilon_t$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

The function $f(t)$ in this setup would represent the potential output and the noise ε_t will represent the output gap.

We would like to model the function $f(t)$ using the truncated polynomial splines as a basis function. Therefore, we would have the following form

$$f(t) = \beta_0 + \sum_{l=1}^{d} \beta_l . t^l + \sum_{k=1}^{K} u_k . (t-\kappa_k)_+^p$$

- d is the degree of the polynomial of time.

- p is the truncated polynomials degree.
- K is the number of knots.
- κ_k is the location of knot k.
  (say at the respective quantiles)
- $(t-\kappa_k)_+^p \equiv \{(t-\kappa_k)_+\}^p$ are the truncated polynomials

For our purpose, we are going to consider $d=1$, and $p=3$. This choice can generate all sort of nonlinear forms.

One question remains, which is the selection of K, the number of knots and of the κ_k, the knot positions. If we choose too big a number of knots, then we might overfit the data. As it is going to be shown shortly, this question has an easy and automatic solution. Briefly speaking, we need to give the model some penalty if we choose too big a number of knots.

We first need to give the mixed model representation of the spline model.

3.3 Linear mixed models interpretation

We can write the model in the matrix form notation

$$\boldsymbol{y} = \boldsymbol{X}.\boldsymbol{\beta} + \boldsymbol{Z}.\boldsymbol{u} + \boldsymbol{\varepsilon}$$

$$\text{Cov}\begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{\varepsilon} \end{pmatrix} = \begin{pmatrix} \sigma_u^2.\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_\varepsilon^2.\boldsymbol{I} \end{pmatrix}$$

- the tth element of $\boldsymbol{y}$ is y_t.
- the tth row of $\boldsymbol{X}$ is $\begin{pmatrix} 1 & t \end{pmatrix}$.
- the (t,j)th element of $\boldsymbol{Z}$ is $(t-\kappa_j)_+^p$.

Therefore, $\boldsymbol{\beta}$ is the vector of coefficients multiplying the trend polynomial and $\boldsymbol{u}$ is the vector of coefficients multiplying the knot truncated polynomials. There is a major difference between $\boldsymbol{\beta}$ and $\boldsymbol{u}$. The first is a vector of fixed coefficients and the latter is a vector of random coefficients. Since this is a model where some of the coefficients are fixed and others are random, it is called a mixed model. Mixed models are a very important class of models. In econometrics, the most important widely used application is the random effects panel data model.

The fact that $\boldsymbol{u}$ is random gives rise to a stringent structure over the coefficients multiplying $\boldsymbol{Z}$. This works as a penalty over those coefficients. If we choose too big a number of knots, this is going to be penalized by the variance of the random effects (i.e. by σ_u^2).

Solving this model can be done in different ways; we can maximize the likelihood $p(\boldsymbol{y}|\boldsymbol{u}).p(\boldsymbol{u})$. This can also be considered (to a certain extent) as minimizing the following objective function

$$\frac{1}{\sigma_\varepsilon^2}.(\boldsymbol{y}-\boldsymbol{X}.\boldsymbol{\beta}-\boldsymbol{Z}.\boldsymbol{u})'(\boldsymbol{y}-\boldsymbol{X}.\boldsymbol{\beta}-\boldsymbol{Z}.\boldsymbol{u}) + \frac{1}{\sigma_u^2}.\boldsymbol{u}'.\boldsymbol{u}$$

Let's introduce the parameter $\lambda^2 = \frac{\sigma_\varepsilon^2}{\sigma_u^2}$. We see that the objective function above can be written as

$$\frac{1}{\sigma_\varepsilon^2}\|\boldsymbol{y} - \boldsymbol{X}.\boldsymbol{\beta} - \boldsymbol{Z}.\boldsymbol{u}\| + \frac{\lambda^2}{\sigma_\varepsilon^2}\|\boldsymbol{u}\|$$

This form can remind us of the objective function for the Hodrick-Prescott filter. Minimizing the first term can be interpreted as minimizing the sum of squared residuals. Introducing he second term in the objective function can be interpreted as allowing a trade-off between the magnitude of the residuals and the smoothness of the curve. (As a matter of fact, if we choose the random effects to be small, then the knot polynomial coefficients will get penalized and the function will be smoother).

Therefore, the parameter λ^2 plays a crucial role in the model. It will tell us how smooth our fit is going to be. Moreover, the choice of λ^2 does not need to be ad-hoc as in the HP filter. As a matter of fact, we can estimate λ^2 from the data!

In addition, if we still want to have control over that parameter, we can consider estimating the model in a Bayesian framework and therefore we can influence the selection of this parameter by putting informative priors on σ_u^2 and σ_ε^2.

3.4 Advantages of penalized splines

Let's summarize the advantages of the penalized spline approach before considering any further questions.

1. It is anchored into a rigorous, widely used statistical model.

   And therefore, statistical inference is possible and is easy to carry out. We can do the following

   - Construct confidence bands for the potential output
   - Or get the whole potential distribution (in case of Bayesian modeling)
   - As well as some forecasting

   Moreover, the choice of λ is no longer ad-hoc and can be estimated from the data.
2. It can have the same interpretations as the HP filter (i.e the loss function interpretation through λ^2).
3. It is not weak at the endpoints (fewer revisions and revisions of a smaller magnitude).
4. No spurious cyclicality, cross- or auto-correlations.
5. It can be easily implemented and undertaken on a routine basis.

The last point is crucial. As explained several times earlier, central bankers need a way that is relatively simple and that can be implemented on a routine basis. For instance, this can be implemented on a routine basis in all the central banks of the European union. And it would not give rise to difficulties of comparison across different EU countries because there is an automatic benchmark against which it is possible to conform. And that benchmark is general enough that it can be easily generalized to more sophisticated settings.

3.5 Estimation

Now that we have discussed the properties of penalized spline models and that we exposed their advantages, we are going to say a few words on estimation.

Estimation is particularly easy. Once one has picked up the knots and constructed the matrices $\boldsymbol{X}$ and $\boldsymbol{Z}$, the estimation is exactly the same as the one for mixed models. From the frequentist perspective, estimation can be carried out using either maximum likelihood (ML) or restricted maximum likelihood (REML). This can be carried quite easily in `R` using the `SemiPar` package. See [Ruppert et al., 2003] and also [Ganguli and Wand, 2005].

From the Bayesian perspective, there is also the WinBugs code constructed partly by the previous authors [Crainiceanu et al., 2005].

More generally, one can estimate the model using one of the algorithms available in [Chib, 1996] and in [Chib and Carlin, 1999].

3.5.1 A note on Bayesian estimation

We are going to illustrate one of the easiest (and less efficient in terms of MCMC mixing) versions.

The unknowns of the model are $(\boldsymbol{\beta}, \boldsymbol{u}, \sigma_u^2, \sigma_\varepsilon^2)$.

Priors

An interesting question is to know whether the fact that $\boldsymbol{u} \sim \mathcal{N}\big(\mathbf{0},\ \sigma_u^2.\boldsymbol{I}_K\big)$ should be considered as part of the likelihood or part of the prior. In Bayesian analysis, this does not matter!

We are considering a hierarchical model.

$$p(\boldsymbol{\beta}, \boldsymbol{u}, \sigma_u^2, \sigma_\varepsilon^2) = p\big(\boldsymbol{u}\big|\sigma_u^2\big).p\big(\sigma_u^2\big).p\big(\boldsymbol{\beta}\big|\sigma_\varepsilon^2\big).p\big(\sigma_\varepsilon^2\big)$$

Again $p\big(\boldsymbol{u}\big|\sigma_u^2\big)$ is going to be considered here as part of the likelihood.

We assume an inverse gamma for both σ_u^2 and σ_ε^2.

$p\big(\sigma_u^2\big)$ is an $\mathcal{IG}_2(s_0^u, \nu_0^u)$.

$p\big(\sigma_\varepsilon^2\big)$ is an $\mathcal{IG}_2(s_0^\varepsilon, \nu_0^\varepsilon)$.

where σ^2 is distributed $\mathcal{IG}_2(s, \nu)$ if the density is proportional to the following expression.

$$\sigma^2 \propto \big(\sigma^2\big)^{-\frac{1}{2}(\nu+2)}.\exp\left\{-\frac{1}{2.\sigma^2}.s\right\}$$

For more details, see [Bauwens et al., 1999].

We assume a normal prior for $\boldsymbol{\beta}$ conditional on σ_ε^2.

$p\big(\boldsymbol{\beta}\big|\sigma_\varepsilon^2\big)$ is a $\mathcal{N}\Big(\boldsymbol{\beta}_0, \sigma_\varepsilon^2.\boldsymbol{\Sigma}_0^{\boldsymbol{\beta}}\Big)$.

Gibbs sampling scheme

A Gibbs sampling algorithm is used. More precisely, we sample from the posterior $p\big(\boldsymbol{\beta}, \boldsymbol{u}, \sigma_u^2, \sigma_\varepsilon^2\big|\mathcal{D}\big)$ by sampling from

1. $p(\sigma_\varepsilon^2|\boldsymbol{\beta}, \boldsymbol{u}, \sigma_u^2)$.
2. $p(\boldsymbol{\beta}|\sigma_\varepsilon^2, \boldsymbol{u}, \sigma_u^2)$.
3. $p(\boldsymbol{u}|\sigma_u^2, \boldsymbol{\beta}, \sigma_\varepsilon^2)$.

4. $p(\sigma_u^2 | \boldsymbol{u}, \boldsymbol{\beta}, \sigma_\varepsilon^2)$.

There are therefore four sweeps inside the Gibbs sampler. We are going to describe how to sample from each one of the full conditionals mentioned earlier.

Conditional on $\boldsymbol{u}$, we have $\boldsymbol{y}^{\boldsymbol{u}} = \boldsymbol{y} - \boldsymbol{Z}.\boldsymbol{u}$.

We can sample successively σ_ε^2 and $\boldsymbol{\beta}$ from the two densities $\mathcal{IG}_2(s_1^\varepsilon,\, \nu_1^\varepsilon)$ and $\mathcal{N}\left(\boldsymbol{\beta}_1, \sigma_\varepsilon^2.\boldsymbol{\Sigma}_1^{\boldsymbol{\beta}}\right)$ where the parameters are

$$\nu_1^\varepsilon = \nu_0^\varepsilon + T$$

$$s_1^\nu = S + s_0^\nu + \left(\boldsymbol{\beta}_0 - \hat{\boldsymbol{\beta}}\right)'.\left(\boldsymbol{\Sigma}_0^{\boldsymbol{\beta}} + (\boldsymbol{X}'.\boldsymbol{X})^{-1}\right)^{-1}.\left(\boldsymbol{\beta}_0 - \hat{\boldsymbol{\beta}}\right)$$

$$\boldsymbol{\Sigma}_1^{\boldsymbol{\beta}} = \left(\left(\boldsymbol{\Sigma}_0^{\boldsymbol{\beta}}\right)^{-1} + \boldsymbol{X}'.\boldsymbol{X}\right)^{-1}$$

$$\boldsymbol{\beta}_1 = \boldsymbol{\Sigma}_1^{\boldsymbol{\beta}}\left(\left(\boldsymbol{\Sigma}_0^{\boldsymbol{\beta}}\right)^{-1}.\boldsymbol{\beta}_0 + (\boldsymbol{X}'.\boldsymbol{X}).\hat{\boldsymbol{\beta}}\right)$$

Note that we have used the OLS quantities

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'.\boldsymbol{X})^{-1}\boldsymbol{X}'.\boldsymbol{y}^{\boldsymbol{u}}$$

$$S = (\boldsymbol{y}^{\boldsymbol{u}} - \boldsymbol{X}.\hat{\boldsymbol{\beta}})'.(\boldsymbol{y}^{\boldsymbol{u}} - \boldsymbol{X}.\hat{\boldsymbol{\beta}})$$

Similarly to sample σ_u^2, It is an $\mathcal{IG}_2$ with parameters $\nu_0^u + T$ and $s_0^u + \boldsymbol{u}'.\boldsymbol{u}$.

Conditional on $\boldsymbol{\beta}$, we have $\boldsymbol{y}^{\boldsymbol{\beta}} = \boldsymbol{y} - \boldsymbol{X}.\boldsymbol{\beta}$. To sample $\boldsymbol{u}$, let's define $\boldsymbol{V} \equiv \boldsymbol{Z}.\sigma_u^2.\boldsymbol{Z}' + \sigma_\varepsilon^2.\boldsymbol{I}_T$ which the variance of $\boldsymbol{y}$ conditional on $\boldsymbol{u}$.

The full conditional of $\boldsymbol{u}$ is proportional to the following quantity

$$\exp\left\{-\frac{1}{2}\left(\boldsymbol{y}^{\boldsymbol{\beta}} - \boldsymbol{Z}.\boldsymbol{u}\right)'.\boldsymbol{V}^{-1}.\left(\boldsymbol{y}^{\boldsymbol{\beta}} - \boldsymbol{Z}.\boldsymbol{u}\right) - \frac{1}{2.\sigma_u^2}\boldsymbol{u}'.\boldsymbol{u}\right\}$$

By completing the square, we find that the full conditional distribution of $\boldsymbol{u}$ is normal with mean and variance

$$\boldsymbol{m}_1 \equiv \boldsymbol{\Sigma}_1^{\boldsymbol{u}}.\boldsymbol{Z}'.\boldsymbol{V}^{-1}.\boldsymbol{y}^{\boldsymbol{\beta}}$$

$$\boldsymbol{\Sigma}_1^{\boldsymbol{u}} \equiv \left(\frac{1}{\sigma_u^2}.\boldsymbol{I}_{d+1} + \boldsymbol{Z}'.\boldsymbol{V}^{-1}.\boldsymbol{Z}\right)^{-1}$$

In order to accelerate mixing, we can use some of the alternative algorithms present in [Chib and Carlin, 1999]. For more details, look at their paper.

3.6 Results

Quarterly GDP data from 1997:Q3 till 2005:Q1 (1997 prices and in Lev).

Results from the spline model (fit using `SemiPar`)

degrees of freedom	λ	number of knots
4.932	8.584	6

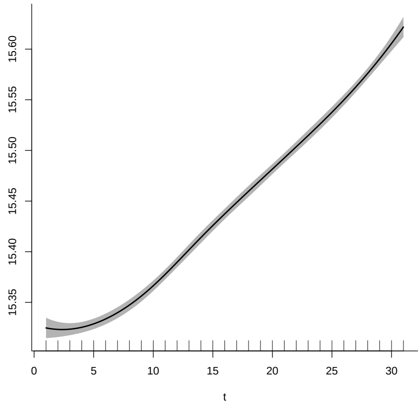

Figure 3. Potential Output Estimates with confidence bands.

The plot of the derivative of potential output with respect to time is quite informative. It gives us some insight about the extent to which the function is nonlinear with respect to time. For details of derivatives estimation, look at [Ruppert et al., 2003] chapter 6.

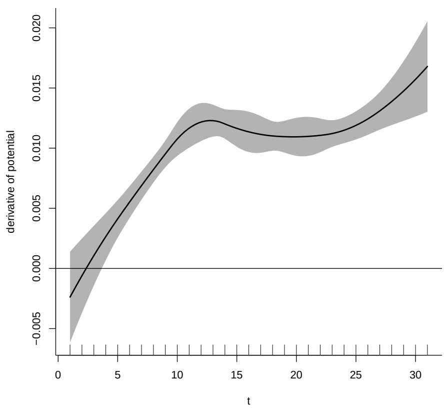

Figure 4. First derivative of potential output with respect to time.

The output gap is just the difference between the output and potential output. We constructed the output gap series with confidence bands.

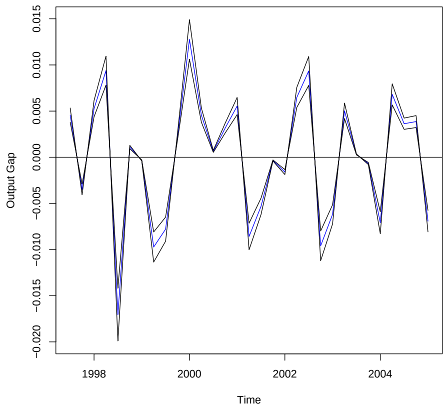

Figure 5. Output gap.

The annual growth rates of potential output is constructed from the quarterly potential output series. Note the similarity between the annual growth rates and the plot of the first derivative of potential output with respect to time. This is another reason why plotting the derivative of potential output with respect to time is a very interesting exercise.

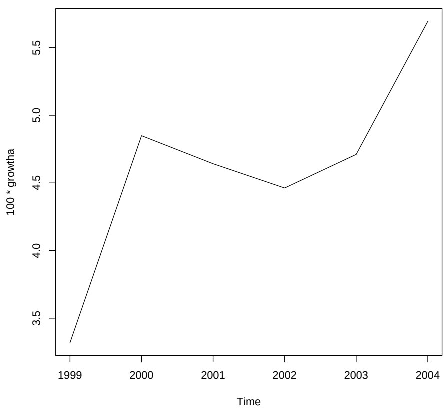

Figure 6. Annual growth of potential output

3.6.1 A rudimentary Phillips curve application

It might be also interesting to examine some applications for the estimation of the output gap series. One such application might be to examine a Phillips curve-like relation for Bulgaria. We undertook the estimation using penalized spline of the following relation.

$$\pi_t = f(g_t) + \varepsilon_t$$

$$\varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$$

where π_t is the inflation rate and g_t is the output gap computed from the earlier exercise. We notice that the relation is somehow nonlinear but this could be there due to some outliers.

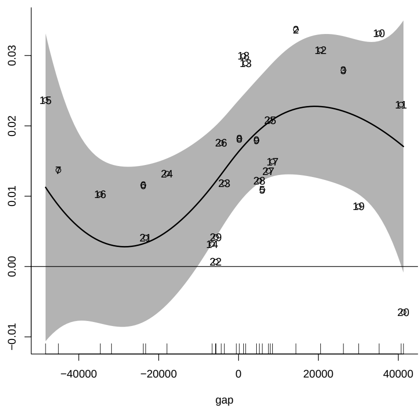

Figure 7. Phillips curve for Bulgaria

3.6.2 Robustness

One way of checking robustness of the model is to do some in-sample forecasting. Cut some points at the end of the sample, fit the potential output model and then try to forecast the potential output for the sample that was cut out. We got some encouraging results using this procedures. Indeed, the forecast estimates of potential output are very close to the estimates of potential output using the whole sample.

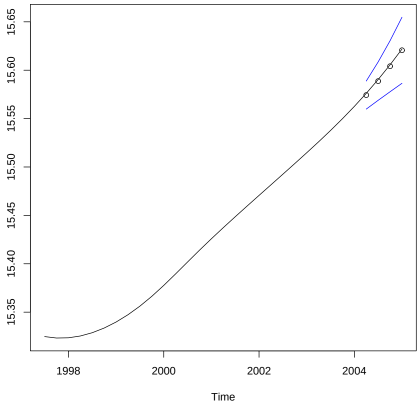

Figure 8. In-sample forecasting. The last four observation were cut out from the sample and then they were forecast. The solid line represents potential output estimated over the whole sample. The dots represent potential output forecast from the small sample and the two lines around the dots represent confidence bands.

3.6.3 A note on forecasting

We are going to apply the methodology of penalized splines to potential output forecasting for the next year.

The forecasts are the following

	2005q2	2005q3	2005q4	2006q1
log potential output forecast	15.63923	15.65781	15.67771	15.69902
standard errors	0.006763034	0.008975445	0.011645387	0.014778245

The table shows that an annual growth rate of 7.94% for potential output is expected for next year (with as low an estimate as 4.9%). However, forecasting using semiparametric models should always be considered carefully.

4 Comparison

Measures of concordance

One way of comparing different estimates from different procedures is to look at some summary statistic describing some sort of concordance. For instance, it might be quite important to check if different procedures yield positive or negative output gap at the same period. This would indicate that usual economic interpretation of overheating or under-performing are the same or not for different series.

For instance, one can think of the variable

$$x_{i,j} = \begin{cases} 1 & \text{if estimate } i \text{ and estimate } j \text{ are both of the same sign} \\ 0 & \text{otherwise} \end{cases}$$

and then, compute the average of such variable.

Using such an estimate of concordance for comparison between different estimates yield the following results.

- Between trend component state-space model and splines estimates
  0.580645161
- Between HP filter and splines estimates
  0.64516129

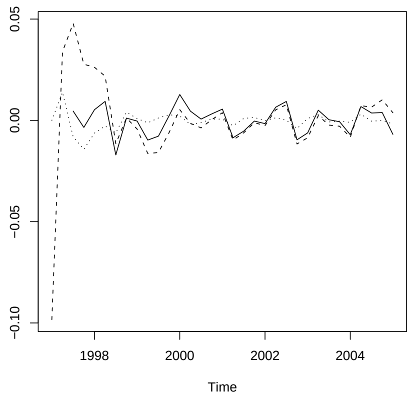

Figure 9. Comparison between three different estimates of potential output.
dashed line: HP filter estimate
solid line: penalized spline estimate
dotted line: estimate through the second-order polynomial trend model.

5 Conclusion

We have demonstrated the use of penalized splines for the estimation of potential output and of output gap. We have showed the numerous advantages of using that methodology over other models. Among these advantages, there is particularly one which is crucial for central bankers and that is the implementability and the reproducibility of results. We hope that this approach is going to be used for such purposes in central banks.

Extension of the approach to other models might be of interest. Mainly, allowing the residuals to have a moving average components (and therefore the output gap to be autocorrelated over time) seems to be an interesting possibility of further research.

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Appendix A The different approaches

This appendix lists the different approaches undertaken for the purpose of potential output estimation.

- The HP filter
- A structural nonlinear and nonnormal state-space approach.
- A polynomial trend linear state-space approach.
- **A penalized spline approach** (adopted by our paper).
- A penalized spline with a moving average component approach.

Appendix B Structural nonlinear and nonnormal state-space approach

We attempted a structural approach for potential output estimation in Bulgaria using a nonlinear and nonnormal state-space model in which capital is taken to be the latent variable. Although we consider the approach to be quite challenging, we abandoned it due to the nature of Bulgaria's labor series. What follows is an outline of the methodology. This might be interesting for further researching the issue!

B.1 Capital as a latent variable

$$Y_t = A.K_t^{\alpha}.L_t^{\beta}.\varepsilon_t$$

$$K_t = I_t + (1-\delta_t).K_{t-1} + \nu_t$$

$$\varepsilon_t \sim \mathcal{L}(0, \sigma_\varepsilon^2)$$

$$\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

$$\text{Cov}(\varepsilon_t, \nu_t) = 0$$

Define $\zeta_t \equiv \log(A) + \beta.\log(L_t)$

The **measurement density** is

$$Y_t \big| K_t \sim \mathcal{L}(\alpha.\log(K_t) + \zeta_t, \sigma_\varepsilon^2)$$

The **transition density** is

$$K_t \big| K_{t-1} \sim \mathcal{N}(I_t + (1-\delta_t).K_{t-1}, \sigma_\nu^2)$$

B.2 Particle filtering

For a reference on particle filtering, see [Pitt and Shephard, 1999].

Purpose

- Approximate the density $p(k_t|D_t)$
  D_t represents information up to time t.
  (i.e. $p(k_t|D_t)$ are the filtered probabilities.)

Initial steps

- Specify M and R.
- Get an initial sample of size M for k_1.

Get for each t a sample $(k^{(m)}, w^{(m)})$, $m = 1, ..., M$.

To proceed from time t to time $t + 1$, use **Sampling/Importance Resampling**

1. Repeat this R times
   Sampling step
   a. Sample a realization of k from the M-sample using the weights (through sampling the index from the uniform distribution).
   b. Generate a $k^{(r)}$ from the transition kernel $K_t|K_{t-1}$.
   c. Compute the likelihood of having obtained Y_t given $k^{(r)}$. Say this probability is $\lambda^{(r)}$.

   Normalize the $\lambda^{(r)}$ and get the new weights $w^{(r)}$.
2. **Importance resampling step**
   Now that we have an R-sample $(k^{(r)}, w^{(r)})$, create an M-sample of realizations of k by using the weights $w^{(r)}$.

B.2.1 The general analysis

- Each M-sample is used as an approximation to the density $p(k_t|D_t)$.

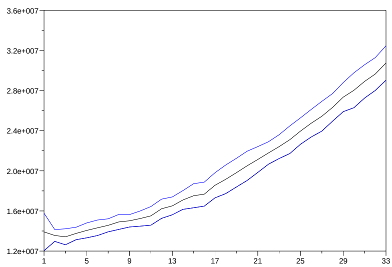

Figure 10. Sampling from the density of filtered probabilities $p(k_t|D_t)$.

B.3 Problems with the structural approach

- Is the right specification or definition of L_t used?
  - Number of employees.

  - Number of hours worked.
  - Value of hours worked.
- There is no relationship between Y_t and L_t.
- Do normal errors in the transition equation represent a good specification?

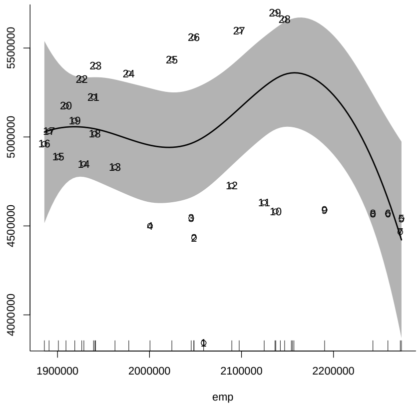

Figure 11. A nonparametric regression of production on employment levels.
This figures explains the empirical problems concerning the relation between Y_t and L_t.

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