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DP/93/2014

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Yavor Kovachev, Daniel Simeonov

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February 2014

ISBN 978–954–8579–53–7

Printed in the BNB Printing Centre.

Elements of the 1999 banknote with a nominal value of 50 levs are used in cover design.

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SUMMARY: This article presents estimates of the German nominal term structure of interest rates over January, 30 2009 – July, 31 2011 period for the purpose of performing a comparative analysis of Bloomberg Fair Value curves. We use the Svensson and Variable Roughness Penalty (VRP) fitting models to generate yield curves for the German bond market for each of the 30 months starting January 2009 and ending July 2011. The key rate yields from the fitted curves which we construct empirically are then extracted and compared with the key rates obtained from Bloomberg to asses the general goodness-of-fit of the two modeling procedures. Technical aspects and underlying assumptions of Svensson, VRP, and Bloomberg Fair Value are described.

Keywords: yield curve modeling, yield curve fitting, variable roughness penaly (VRP), cubic splines, Nelson-Siegel model, Svensson model, Bloomberg Fair Value curves.

At the time the preparation of the paper Yavor Kovachev was an intern at the Bulgarian National Bank and a Ph.D. student of Financial Mathematics at "Florida State University" and Daniel Simeonov was a risk analyst at the Bulgarian National Bank, and later – visiting professor at Sofia University "St. Kliment Ohridski."

The opinions presented in the paper are those of the authors and should not be associated with the institutions in which they worked.

The authors would also like to provide their special thanks to Ivan Minchev, Head of Risk Analysis at the Bulgarian National Bank, for insightful discussions on and the comments and feedback received at different stages of preparing this work.

The authors would also like to ackowledge the editing contribution of Nickolay Nedelchev, an associate in DCH LLC and a visiting professor at Sofia University "St. Kliment Ohridski," during the final stages of this paper.

Practical and theoretical significance of accurate yield curve models

Designing accurate methods for modeling the term structure of interest rates is essential for all market participants. As indicators of market expectations of future inflation, yield curve estimates serve as a tool for assessing financial markets' perception of current and future economic conditions. Thus, in their dual role of forecasting inflation and gauging perceptions of economic stability, accurate yield curve estimates are essential tools for central banks attempting to conduct effective monetary policy and for efficient reserve management. Additionally, the accurate pricing of virtually any financial instrument depends on obtaining accurate estimates of the implied forward interest rates, which is key for both strategic and tactical asset allocation.

To obtain estimates which are useful and "accurate," a good model should generally possess three basic characteristics: smoothness, flexibility and stability. Different models provide different trade-offs between the three characteristics but all three are crucial for effective modeling. Smoothness is essential for monetary policy since smooth models try to capture the general trends in the implied forward rates of interest and provide central banks with a clearer picture of market perceptions. On the other hand, less smooth models are useful for financial practitioners as they try to incorporate any pricing idiosyncrasies resulting from market inefficiencies or shocks in economic activity. Flexibility is necessary for capturing movements in the term structure both from a monetary policy and practical perspective. Finally, stability is important in ensuring that changes in estimates at a specific maturity range of the forward curve do not adversely and inexplicably affect estimates for the rest of the maturity domain.

Technical specifications of the estimation methods

We investigate two alternative methods for estimating the term structure of interest rates. Svensson's (1994, 1995) parametric model hereafter SV, and Waggoner's (1997) variable roughness penalty model based on cubic splines hereafter referred to as VRP. An overview of the basic characteristics of the two models is provided in Table 1:

Table 1

OVERVIEW OF MODELS

Property	Svensson	VRP
Type of model	Parametric	Spline-based
Nature of forward rate curve	Forward rate is a function defined for all maturities	Forward rate is a piece-wise function where individual pieces are quadratic polynomials joined at predefined knot points
No. of parameters	6	Depends on number of instruments/bonds used
Objective function	Minimize residual sum of squares	Minimize residual sum of squares plus roughness penalty
Pre-specified parameters	None	Number of knot points, smoothing function
Constraints	Long-run asymptote	None

Both methods model the forward rate curve as a function of a number of pre-specified parameters. However, they present distinct functional specifications for the term structure based on the differing criteria used for determining the optimal estimates of the parameters. These differences result in specific trade-offs between the flexibility or goodness-of-fit and the smoothness of the final functional form which will be discussed in further detail in the following sections.

Svensson's model

Svensson's model is an extension of the parsimonious parametric Nelson-Siegel (NS) model introduced in 1987. The unifying idea of parametric models is the construction of a single i.e. one-piece function defined over the entire maturity domain. For the NS model the instantaneous forward curve is defined as follows:

$$f_{t,m}(m) = \beta_{t,0} + \beta_{t,1}\exp(-\frac{m}{\tau_{t,1}}) + \beta_{t,2}\frac{m}{\tau_{t,1}}\exp(-\frac{m}{\tau_{t,1}}) \quad (1)$$

where *m* denotes time to maturity, *t* is the time index and the remaining four variables, β_0, β_1, β_2 and τ_1, are the parameters to be estimated. To simplify notation the index *t* is dropped. Deriving the spot rate curve is done by using the fact that the yield to maturity is given by the average of the forward rates i.e.

$$s_m(m) = \frac{1}{m}\int_0^m f_m(x)\mathrm{d}x \quad (2)$$

Integrating (1) results in:

$$s_m(m) = \beta_0 + \beta_1\left(1 - \exp(-\frac{m}{\tau_1})\right)\left(\frac{m}{\tau_1}\right)^{-1} + \beta_2\left(\left(1 - \exp(-\frac{m}{\tau_1})\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp(-\frac{m}{\tau_1})\right) \quad (3)$$

Grouping and rearranging:

$$s_m(m) = \beta_0 + (\beta_1 + \beta_2)\frac{\tau_1}{m}\left(1 - \exp(-\frac{m}{\tau_1})\right) - \beta_2\exp(-\frac{m}{\tau_1}) \quad (4)$$

At its long end the NS curve (3) asymptotically approaches the value of which must be positive. Thus, the NS curve is constrained to converge to a constant level for maturities in the 20 to 30 years range. The constraint is introduced in the model to reflect the fact that long-term forward rates are derived on the basis of expected future short-term interest rates with the addition of a constant risk premium. This is equivalent to satisfying the expectation hypothesis. The sum $\beta_0 + \beta_1$ determines the starting value of the NS curve at maturity *0*. Thus β_1 can be interpreted as the deviation from the long run interest rate. The remaining two parameters β_2 and τ_1 are responsible for the hump of the interest rate curve. The magnitude of the hump is given by the absolute value of β_2 , while the sign of the parameter determines its direction i.e. a negative sign on β_2 results in a U-shape and a positive sign in an inverted U-shape or a hump. Finally, the position of the hump is given by τ_1 (BIS 2005).

Svensson's model (SV) proposed in 1994 is an extension of the NS model and introduces two additional parameters β_3 and τ_2. These parameters add additional flexibility to the interest rate curve as they introduce a second hump to the functional form. The updated SV instantaneous forward rate then becomes:

$$f_m(m) = \beta_0 + \beta_1 \exp(-\frac{m}{\tau_t}) + \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) + \beta_3 \frac{m}{\tau_2} \exp(-\frac{m}{\tau_2}) \quad (5)$$

where β_3 and τ_2 have the same characteristics as β_2 and τ_1 and discussed above. Spot rates are derived in a similar fashion by integrating the instantaneous forward rate curve which results in the following functional form:

$$s_m(m) = \beta_0 + \beta_1 \left(1 - \exp(-\frac{m}{\tau_1})\right)\left(\frac{m}{\tau_1}\right)^{-1} + \beta_2 \left(\left(1 - \exp(-\frac{m}{\tau_1})\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp(-\frac{m}{\tau_1})\right) + \\ + \beta_3 \left(\left(1 - \exp(-\frac{m}{\tau_2})\right)\left(\frac{m}{\tau_2}\right)^{-1} - \exp(-\frac{m}{\tau_2})\right) \quad (6)$$

In the case of the SV model the objective function we want to minimize is:

$$\min \sum_{i=1}^{N} \left(\frac{P_i - \hat{P}_i(f_m)}{D_i}\right)^2 \quad (7)$$

where P_i is the observed price of i-th bond, D_i is the i-th bond's duration and $\hat{P}_i(f_m)$ is the fitted price derived from the estimated forward curve.

The Variable Roughness Penalty (VRP) model

Rather than specifying a single functional form over the entire maturity domain, general spline-based fitting methods model the instantaneous forward rate curve with piecewise n-th degree polynomials joined at predetermined knot points. In practice, the most commonly used polynomials are cubic polynomials. The main idea is to fit a piecewise function of the form:

$$S(x) = \begin{cases} s_1(x) & for \quad x_1 \le x < x_2 \\ s_2(x) & for \quad x_2 \le x < x_3 \\ \dots \\ s_{n-1}(x) & for \quad x_{n-1} \le x < x_n \end{cases} \quad (8)$$

where s_i is a third degree polynomial defined by

$$s_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \quad (9)$$

for i=1,2,...,n-1.

and $s_i(x)$ has to satisfy:

1. $S(x)$ interpolates all bond data
2. $S(x)$ is continuous on the interval $[x_1,x_n]$
3. $S'(x)$ is continuous on the interval $[x_1,x_n]$
4. $S''(x)$ is continuous on the interval $[x_1,x_n]$

Under these conditions, both the curve and its first and second derivatives are continuous for all maturities. Since the individual piece-wise polynomials are allowed to move almost independently from each other, subject to the continuity constraints, this method allows for a much higher degree of flexibility than the parametric NS and SV models. This is the main advantage that spline-based models have over parametric models for curve fitting, i.e. individual segments of the interest rate curve will not be severely affected by increased interest rate volatility in nearby segments whereas under parametric models a change in one data point can affect the entire curve.[1]

In general, however, traditional cubic spline-based techniques can be too flexible to generate yield curves with the appropriate level of smoothness required for market pricing or monetary policy purposes. In order to tackle this problem, Fisher, Nychka and Zervos (1995) (FNZ) develop a method commonly known in the academic literature as "smoothing splines". For this method a large number of knot points are included in the objective function in order to allow for a sufficient degree of flexibility in the curvature of the spline function. The optimal number of knot points is then determined by minimizing the ratio of goodness-of-fit to smoothness, where smoothness is given by the second order derivatives $s''(x)$. Thus, this approach is constructed to penalize the presence of parameters which do not contribute significantly to the overall fit of the spline function.

There exists a large number of spline based methodologies which use the smoothing techniques pioneered by FNZ. The main difference among those is the manner in which the smoothing criteria are applied to reach higher goodness-of-fit. The general specification of the objective function is:

$$\min \sum_{i=1}^{N} \left(\frac{P_i - \hat{P}_i(f(m))}{D_i}\right)^2 + \int_0^M \lambda_t(m)[f''(m)]^2 \mathrm{dm} \qquad (10)$$

where the first term is the same as with the NS and SV models and the second term represents the roughness penalty. Here $f''(m)$ is the second

[1] See Anderson and Sleath (2001) for an analysis and illustration of how the Svensson model curve can be severely affected by changes in single data points.

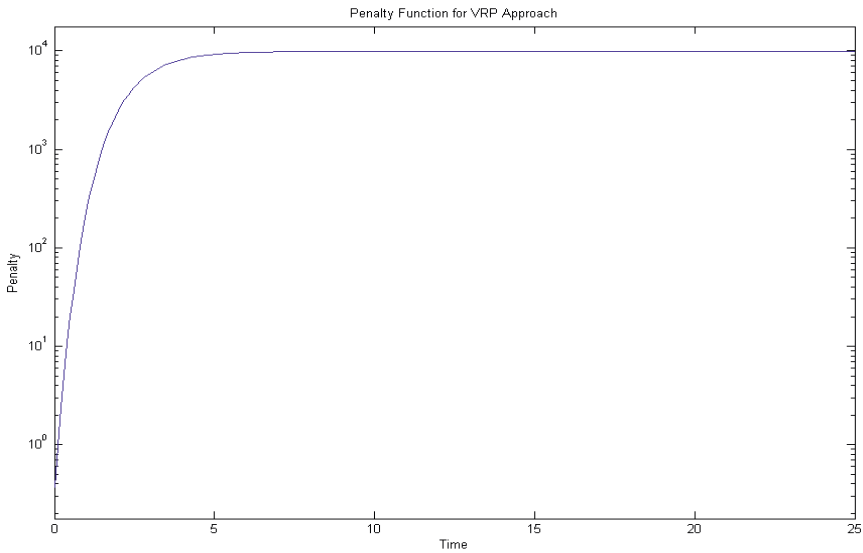
derivative of the forward rate curve and is the measure of its curvature, while $\lambda_t(m)$ is the penalty function. As the name suggests, the purpose of this function is to penalize excessive curvature i.e. non-smoothness in the forward rate curve. Different functional specifications for $\lambda_t(m)$ can be utilized, a constant function or a step function proposed by Waggoner are two options. The specification which we have chosen to employ comes from the VRP approach developed by Sleath and Anderson (1999) at the Bank of England. The $\lambda_t(m)$ function they specify is given by:

$$\log(\lambda_t(m)) = L - (L - S)\exp(-\frac{m}{\mu}) \tag{11}$$

Here the roughness penalty function is allowed to vary with maturity permitting more curvature at the short end and limiting curvature at the long end. Graphically:

Figure 1

VRP PENALTY FUNCTION



The Bloomberg Fair Value curves – overview

The Bloomberg Fair Value curves are used as an off-the-shelf solution, which is compared both to the Svensson and VRP models. A practical aspect of the present work is to compare the reliability of direct use of the zero-coupon curves available directly off-the-shelf to Bloomberg users as alternative to in-house modeling.

Below we summarize the key points regarding Bloomberg Fair Value curves as presented by Bloomberg L.P. at the International Bond Market Conference in Taipei 2007.[2]

A BFV price is a model or derived price of a bond. It indicates where the price of a bond *should* trade based on where comparably rated bonds with comparable maturities actually trade. It is derived by utilizing well-priced bonds with similar characteristics i.e., currency, market type, industry, and credit rating. Only bonds with Bloomberg Generic (BGN) prices are included.[3] In some cases, bond prices from a specific pricing source are used in lieu of BGN prices (e.g. fixing prices).

A yield curve is built daily for each sector based on the population of bonds directed to that sector or curve. As the model derives a price for a bond based on the BFV market curve, any bonds will have a BFV price without the bond's physical presence on a sector curve.[4]

The zero yield curve is modeled and all other curves (par coupon curve and forward curve) are derived from the zero curve. A piecewise linear function is used to estimate the zero coupon yield curve .

The following constraints are imposed:

Pure parameters constraints: the upper and lower bounds imposed on each parameter to ensure the optimization engine would work properly

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[2] The original presentation provides in-depth discussion on assumptions, challenges, and some historical items in the evolution of the model, whose roots date back to 1995. The account presented below does not cover further evolutions of the model, following the 2007 presentation by Bloomberg L.P.

[3] BGN price is the simple average price of all kinds of prices, including indicative prices and executable prices, quoted by our price contributors over a specified time window. The availability of BGN price for a bond is an indication of good liquidity for that bond. In some cases, bond prices from a specific pricing source are used in lieu of BGN prices (e.g. fixing prices). Outliers (i.e. bonds whose OAS are significantly higher or lower than OAS of comparable bonds) are excluded.

[4] The advantage of a piecewise model is that it contains more points (parameters) than parameterized smooth curve (which only contains 4-5 parameters). More parameters can better fit a sector yield curve. One disadvantage of piecewise function is that it could result in unstrippable zero curves and negative forward rates as well. The smoothed curve function , though never generates unstrippable curve and always has positive forward rates, cannot in general fit the sector curve as well as a high parameter piecewise linear curve. An additional disadvantage of the smoothed curve function is that, when the curves have complex parameterizations, it is hard to analytically specify the cross sector constraints, and thus hard to enforce them in the optimization of the curves.

Yield constraints: (1) Credit constraints: *Forward curve* $f_i(T) > f_j(T)$ *if credit sector i is risk than j for any* $T \geq 0$*; (2)* Positive forward rates constraints: *Forward curve* $f_i(T)$ *should be positive for any sector I for any* $T \geq 0$*; (3)* Technical constraints: *Zero curve* $Z_i(T)$ *must be in the functional set suitable for the current bond price – OAS pricing engine; (4) Continuity constraint: the yield curve is continuous at all points on the curve, including the yields adjacent to the maturity points.*

The interest rate model used in the yield curve modeling is the lognormal model. Any interest rate options embedded in callable and puttable bonds are valued with the lognormal interest rate model and zero curve $Z_i(T)$ and yield volatility σ are the parameters of the lognormal interest rate model.

The bonds within the same credit sector are broken down into zero coupon cash flows and a constant OAS spread is added to the zero curve $Z_i(T)$ to generate a set of discount rates which are used to discount cash flows of any bond into its present value. The present value of a bond's future cash flows is the estimated market value of that bond.

For any given set of zero curve $Z_i(T)$, OAS_i is determined such that the estimated market value of the bond i is equal to the BGN full price of that bond. If a bond has embedded option, as in a callable bond or puttable bond, both implied volatility and ΔOAS are determined for the bond. The optimized zero curve, or the best fitted zero curve, is the curve that minimizes the sum of squares of OAS_i. Par coupon curves are derived from the optimized zero curves.

Data

End of month data was collected from Bloomberg for the following bond indices: EGB0, EG0A, G5D0 and G9D0, all constructed by Bank of America Merrill Lynch. The beginning date used is January, 30 2009 and the end date is June, 30 2011. The two bond indices EGB0 and EG0A were combined and only the German bills and bonds were extracted to construct the G00D[5] index of Germany for the maturity range of 0 to 1 years. G5D0 is a standard Bank of America Merrill Lynch index which includes German bonds which fall in the 1 to 10 years maturity range. Similarly, G9D0 is a standard Bank of America Merrill Lynch index which encompasses the 10 to 30 years maturity range. The data from the three indices was combined and the resulting instruments sorted by maturities from 0 to 30 years. Since both the

[5] G00D is not an off-the-shelf index. It was created simply to hold German bills and bonds with maturities in the 0 to 1 years range.

SV and VRP model were estimated using Matlab, additional information on the instruments' coupon rate, maturity, period, basis, end-month-rule, first coupon date, clean price, accrued interest, issue date and accrual start date was collected from Bloomberg. Period here refers to the periodicity of the coupons of the bonds in the 30 samples. All of the bonds included have annual coupon payments and therefore a periodicity of 1.[6] Last coupon date, start date and face value are taken to be the default which in Matlab means last coupon date equals maturity, start date equals settle date i.e. the last working day of the month for our 30-month sample and face value equals 100. Basis refers to the day count basis for the bonds and the basis used here is actual/360 for BUBILLs and actual/actual for the remainder of the instruments (BKO, OBL, DBR). For BUBILLs the actual issue date obtained from Bloomberg was used; for the remainder of the instruments the start accrual date was used instead of the issue date to ensure that Matlab generates the right cash flow dates for the coupons. Finally, the cfamounts function in Matlab was used to generate the cash flow in the 30-month sample. To ensure the correct cash flows were generated, the data from Matlab were compared with actual historic cash flow data obtained from Reuters. The conclusion was that the constructed cash flows are identical to actual historic cash flows. Very few minor chronological discrepancies of 1 or 2 days were observed for coupon payments that fall on Saturday or Sunday respectively. This is due to the fact that, by convention (modified following), coupons for German bonds whose cash flows fall on weekends are paid on the next working day which is either one or two days in the future, depending on whether the actual cash flow date is on a Sunday or a Saturday respectively.[7]

Results

Graphical analysis

Yield curves were fitted with SV and VRP for all 30 data sets. Results are similar across the board as can be seen from the detailed statistical analysis table below and one date is chosen to show the graphical comparison between SV, VRP, and Bloomberg fair Value curves.

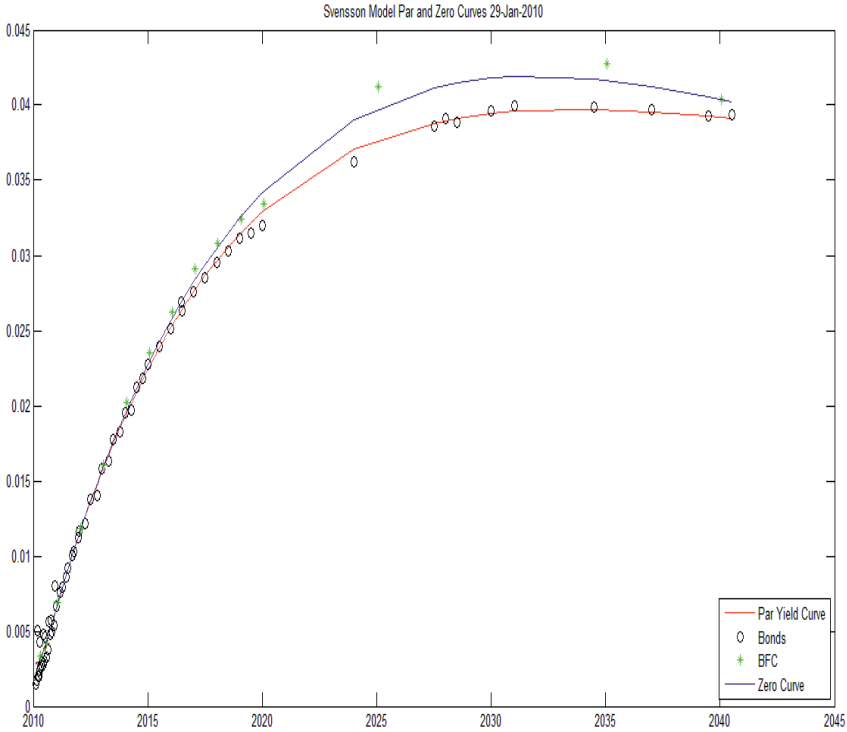
Shown below in Figure 2 and Figure 3 are the results when the SV model is fitted to data with settle date 29-Jan-2010.

[6] For a review of the instrument related conventions used in Matlab refer to http://www.mathworks.com/help/toolbox/finfixed/fitsvenssonirfunctioncurve.html

[7] Only if the Saturday is the last day of the month, the coupon is paid on the preceding Friday.

Figure 2

SVENSSON MODEL WITH PAR YIELD AND ZERO CURVE



Notice here that the par yield curve interpolates the sovereign bond data and the zero curve needed for pricing policy and investment decisions is close to the Bloomberg fair value curve data particularly at the shorter maturity domain. To see the result of the fitting method more clearly we can remove the scatter for the bond maturities and their respective yields. The result is shown in Figure 3.

Figure 3

SVENSSON MODEL WITH ZERO CURVE ONLY

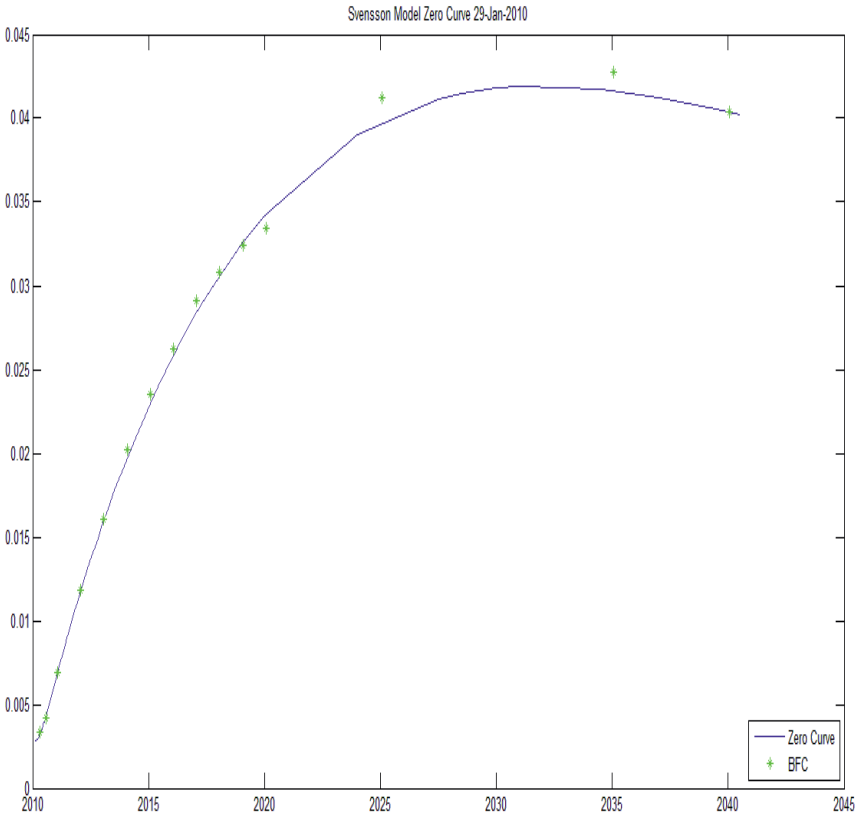
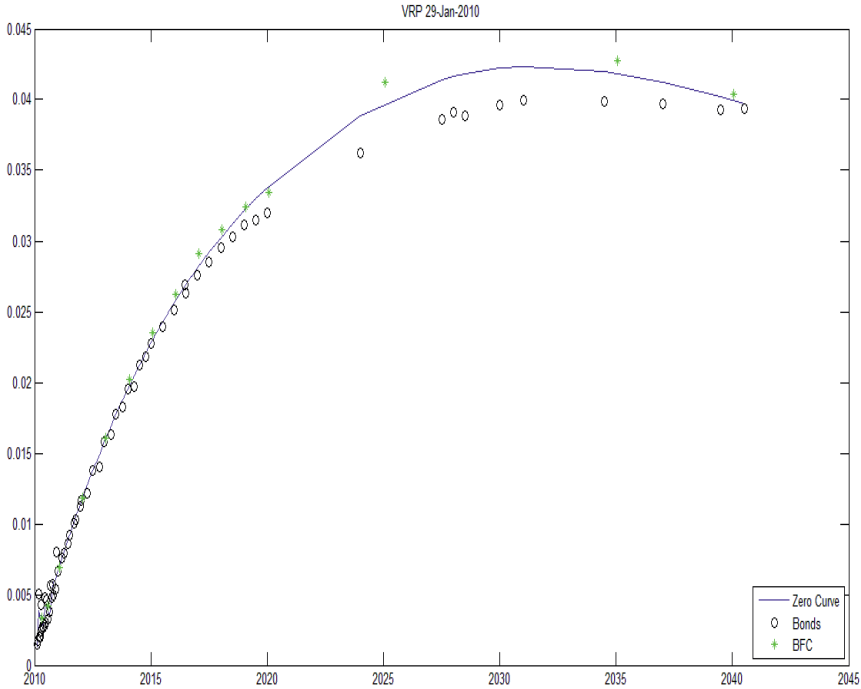


Figure 3 displays the zero curve obtained with SV for all the bonds in the three indices G00D, G5D0 and G9D0 with the settle date 29-Jan-2010 and maturity horizon of 30 years. The green points are the key rates obtained from Bloomberg for the same settle date. The scatter of the Bloomberg fair curve maturities and yields has been added to the data file after the SV fitting algorithm has been carried out, i.e. the blue zero curve has been generated before green stars were added to the data plot.

Similar graphical results obtained with VRP for the zero interest rate curve are displayed in Figure 4.

Figure 4

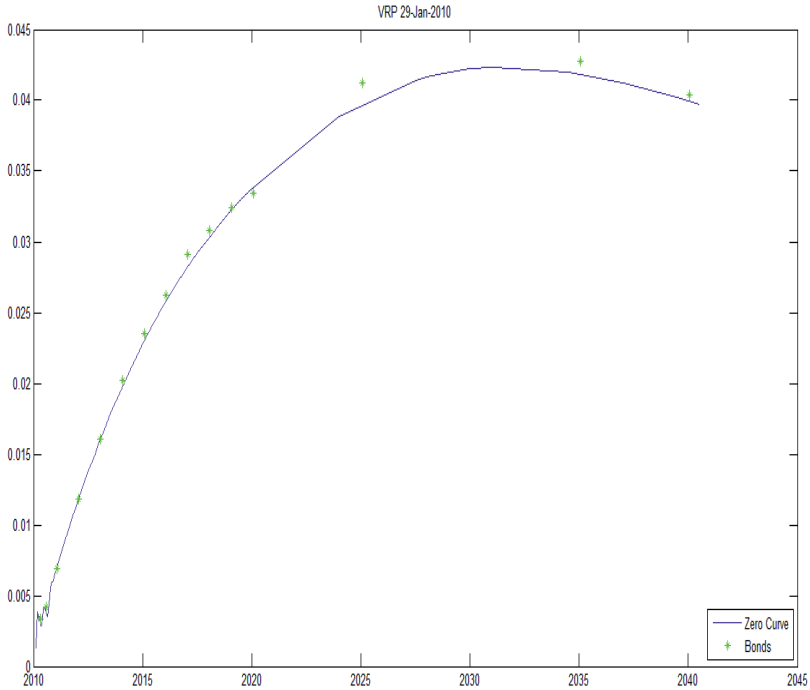
VRP MODEL WITH ZERO CURVE AND BOND DATA POINTS



When the scatter of bond maturities and yields is removed we get the result displayed in Figure 5. As previously explained, the legend in the lower right corner of the plot contains the zero curve generated from all three indices for the settle date 29-Jan-2010 along with the actual maturity-yield combinations for the bonds and, with the green stars, the key rates for the Bloomberg fair value curve with the same settle date.

Figure 5

VRP MODEL WITH ZERO CURVE ONLY



It is important to notice the yield curve obtained with the VRP model contains a sharp jagged saw-tooth section in the short end. This is mainly due to the specification of the penalty function that was chosen. Unlike the SV model whose functional specification imposes smoothness, the VRP model captures the interest rate volatility at the short end of the yield curve and is therefore inferior as a pricing tool for practitioners. As was illustrated in Figure 1, the $\lambda_t(m)$ specification for this VRP model is chosen so that it does not penalize curvature at the short end, which leads to the saw-tooth regions appearing at the very short ends of the estimated yield curves.

Statistical analysis

To assess the goodness-of-fit of the obtained yield curves from the two fitting methods we use three different measures of statistical fit, namely, the mean squared error (MSE), the root mean squared error (RMSE) and the Theil U-statistic. The formulae for the three statistical tests are given below for reference:

$$\mathrm{MSE} = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{n}}$$

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t)^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t)^2}}$$

The score returned by the Theil test varies from 0 to 1, with 1 meaning *maximum disagreement* between the fitted values obtained with the two models and the actual fair value key rates given by Bloomberg.

The results of the three statistical tests are summarized for each of the two fitting procedures in Table 2.

Table 2

MSE, RMSE AND THEIL STATS FOR THE SV AND VRP

	Svensson			VRP		
	MSE	RMSE	Theil	MSE	RMSE	Theil
Average	1.15E-06	9.60E-04	1.83E-02	6.743E-07	7.713E-04	0.013945

	Svensson			VRP		
Starting date	MSE	RMSE	Theil	MSE	RMSE	Theil
6/30/2011	3.713E-07	0.0006094	0.011378	3.08E-07	0.000555	0.009706
5/31/2011	3.637E-07	0.0006031	0.011532	8.24E-07	0.000908	0.016365
4/29/2011	3.722E-07	0.0006101	0.010889	4.03E-07	0.000634	0.010704
3/31/2011	3.817E-07	0.0006178	0.011788	4.6E-07	0.000678	0.012066
2/28/2011	4.587E-07	0.0006773	0.012727	3.67E-07	0.000606	0.010749
1/31/2011	5.088E-07	0.0007133	0.013466	4.12E-07	0.000642	0.011435
12/31/2010	6.301E-07	0.0007938	0.016446	3.8E-07	0.000616	0.011978
11/30/2010	5.888E-07	0.0007673	0.017476	4.06E-07	0.000637	0.01361
10/29/2010	1.074E-06	0.0010365	0.023781	6.6E-07	0.000813	0.017528
9/30/2010	4.581E-07	0.0006768	0.017522	4.59E-07	0.000678	0.016325
8/31/2010	1.695E-07	0.0004117	0.011752	2E-07	0.000448	0.011868
7/30/2010	4.729E-07	0.0006877	0.015333	5.03E-07	0.000709	0.014679
6/30/2010	9.718E-07	0.0009858	0.023124	5.26E-07	0.000725	0.015734
5/31/2010	3.74E-07	0.0006115	0.01399	2.12E-07	0.00046	0.009727
4/30/2010	2.796E-07	0.0005288	0.010597	2.19E-07	0.000468	0.00872
3/31/2010	8.191E-07	0.0009051	0.017271	4.58E-07	0.000677	0.012039
2/26/2010	3.893E-07	0.0006239	0.011878	4E-07	0.000632	0.011222
1/29/2010	3.50E-07	0.0005916	0.010867	3.43E-07	0.000586	0.010057
12/31/2009	5.647E-07	0.0007515	0.012963	8.43E-07	0.000918	0.01485
11/30/2009	6.655E-07	0.0008158	0.014992	3.79E-07	0.000615	0.010594
10/30/2009	8.888E-07	0.0009427	0.016672	4.37E-07	0.000661	0.010969
9/30/2009	8.621E-07	0.0009285	0.016626	6.66E-07	0.000816	0.013714
8/31/2009	8.621E-07	0.0009285	0.016626	6.66E-07	0.000816	0.013714
7/31/2009	9.709E-07	0.0009853	0.017314	4.51E-07	0.000672	0.011073
6/30/2009	2.631E-06	0.0016222	0.027376	1.19E-06	0.001092	0.017243
5/29/2009	2.913E-06	0.0017066	0.027414	7.81E-07	0.000884	0.013344
4/30/2009	5.215E-06	0.0022836	0.041433	3.61E-06	0.0019	0.032426
3/31/2009	3.851E-06	0.0019623	0.036123	1.4E-06	0.001181	0.02044
2/27/2009	4.256E-06	0.0020631	0.037368	1.42E-06	0.00119	0.020376
1/30/2009	1.84E-06	0.0013565	0.023453	8.5E-07	0.000922	0.015101

Conclusion

The statistical results presented in Table 2 along with the graphical results presented in Figures 2 through 5 show that the yield curves constructed with the Svensson and the Variable Roughness Penalty approaches provide a close estimate to the key rates obtained from Bloomberg. This shows that Bloomberg Fair Value curves are reliable off-the-shelf solution. Thus, we can conclude that those yield curves are suitable models of the actual term structure of interest rates in Germany and can, fairly safely, be used as reliable tools for pricing financial instruments with maturities that fall between the key rates.

The statistical measures of fit indicate that the Variable Roughness Penalty method generally outperforms Svensson's model as technique for obtaining fair value key rates. However, VRP is less useful from a practical standpoint as it captures interest rate volatility at the short end of the yield curve. Therefore, Svensson's model is preferred to VRP with the given $\lambda_t(m)$ specification as the model of choice for both the pricing of financial instruments and the analysis of monetary policy.

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