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Abstract: This paper studies how macroeconomic and fiscal shocks affect Bulgarian public debt-to-GDP ratio. A quarterly VAR model with debt feedback is used to assess the effect of different shocks on debt dynamic through impulse response functions and debt decomposition. The main finding of this study is that fiscal consolidation leads to a decline in public debt that is statistically significant in the first two and a half years following the shock. The debt ratio then returns to its pre-shock path. An inflation shock somewhat reduces the debt ratio for the first few quarters as well. A positive growth shock leads to a statistically significant debt reduction in the following 5 years. In the current specification, the debt level is stationary, whereas a VAR which excludes debt may lead to an explosive debt path.

Keywords: Public debt, VAR, fiscal policy, impulse response

JEL classification numbers: H60, E31, E62, C32

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1. Introduction

The global economic and financial crisis left a legacy of historically high levels of public debt which rose considerations about public debt sustainability. In the period 2007–2014 public debt-to-GDP ratio increased by 26 percentage points for the EU-28 with the highest increases ranging from around 80.6 percentage points of GDP in Ireland to more than 50 percentage points of GDP in Greece, Spain, Portugal and Slovenia. Several EU countries took actions in an attempt to reduce fiscal balances and to improve their creditworthiness, which led to a reversal of the increasing debt ratio path in 2015 on EU-level. Sustainable public finances and low level of public debt in particular, provide countries with fiscal space to counter negative macroeconomic developments over the business cycle. In long term prospective, ensuring sustainable public finances creates fiscal space for governments to cope with implicit liabilities, such as aging costs. A broad set of policy options for public debt reduction include, fiscal consolidation, growth-enhancing policies, higher inflation or a reduction of interest rates. Nevertheless, the main challenges facing policymakers are to choose the optimal timing, pace, and tools to reduce public debt.

Conducting restrictive fiscal policies in an effort to get debt ratio converge to the official target could be a drag on economic growth and lead to an increase rather than a decrease in debt-to-GDP ratio. On the other hand, pursuing policies supporting growth could positively weigh on debt dynamics and improve fiscal budget balance bringing more budgetary revenues. Higher inflation is another channel to reduce high debt levels. An increase in inflation would decrease the real value of debt but to have a lasting impact on the debt inflation would need to be combined with financial repression such as regulations on cross-border capital movements and explicit or implicit caps on interest rates (Reinhart and Sbrancia, 2011). Nonetheless, a radical change in monetary policy would be required in case a higher inflation target is allowed.

The aim of this paper is to investigate the effects of austerity, inflation, and growth shocks on reducing public debt-to-GDP ratio for the Bulgarian economy in order to assess whether debt reduction episodes tend to be short-lived or more persistent over time. To be able to study the relationship between debt dynamics and macroeconomic and fiscal variables, a Vector Autoregressive (VAR) model with quarterly data for the period of 2000:Q2 – 2019:Q4 is used. The VAR framework includes four endogenous variables (GDP growth, the primary budget balance, the inflation rate and the government implicit interest rate) and two lags of debt-to-GDP ratio as an exogenous variable. At the same time, the government budget constraint is included as a separate equation. The VAR framework bears a close resemblance to the model proposed by Favero and Giavazzi (2007). The main advantage of estimating a debt-augmented VAR model is that it allows tracing out debt dynamics *via* the deterministic debt equation, while allowing the endogenous variables in the VAR to respond to the level of the debt ratio along the path induced by the fiscal shock. Omitting the debt feedback from the debt level to changes in the endogenous fiscal variables could lead to the calculation of biased coefficients which are then used in the computation of the impulse response functions. As pointed out by Favero and Giavazzi (2007), such a bias, in turn, could result into impulse responses, which are sometimes computed along unstable debt paths, *i.e.* paths along which the debt-to-GDP ratio diverges. The results from this model, applied to the Bulgarian economy, show that a VAR framework with debt feedback results into a debt ratio that is stationary, whereas VARs excluding debt feedback could lead to an explosive debt path. To my knowledge, this is the first study to apply a debt-augmented VAR approach to a country from Central and Eastern Europe and Bulgaria in particular that considers the effect of different macroeconomic and fiscal shocks on public debt dynamics. The explicit reconstruction of the debt dynamics following fiscal, growth and inflation shocks is another important contribution of the paper. In order to check the validity of the proposed debt augmented VAR framework for the Bulgarian economy, the debt series obtained through the deterministic debt equation are compared to the historical debt values. In most of the cases the simulated series follow quite close the actual debt data which allows the application of the selected methodology.

The main finding of the analysis is that, if the economy continues to behave as in the recent past (2001–2019), given the simulated shocks in this study public debt-to-GDP ratio decreases after an episode of fiscal consolidation, positive growth shock and higher inflation and then returns to its pre-shock level. The general view in the literature supports the idea that in recessionary time, when fiscal multipliers are larger than in normal times, fiscal consolidation might result in an increase rather than a decrease in debt ratio. However, in small open economies there is a broad consensus that fiscal multipliers are higher during periods of economic recession, but they are typically small. The non-linear relationship between macroeconomic variables and public debt ratio, presented in the current VAR framework, could imply that the economic environment may matter. Changes in the initial economic conditions lead to higher debt reduction under times with more depress economic conditions and elevated debt ratios. Debt reduction following fiscal consolidation and a higher inflation shock seems to be not statistically significant after around three years whereas a growth shock leads to a higher and more persistent debt reduction that is statically significant throughout the next 5 years.

The paper is structured as follows. The next section discusses the main concepts behind public debt sustainability and provides a review of fiscal VAR literature, section 3 describes some structural aspects of the Bulgarian economy, section 4 provides details on the estimation methodology used to analyze public debt dynamics under different macroeconomic shocks. Section 5 describes the data. Section 6 presents the empirical results related to the baseline specification. Section 7 illustrates the results obtained first from a VAR model that omits the debt feedback and then a second model applying the structural approach of Blanchard and Perotti (2002). Section 8 describes a set of conclusions and some policy implications.

2. Theoretical Framework and Related Literature

Public debt sustainability means that the accumulated debt has to be serviced at any point in time. This implies that government must be both liquid and solvent. Liquidity is a short-term concept and refers to government's ability to maintain access to financial markets to service its obligations. Thus, for public debt to be sustainable, a country should be able to preserve its market access in the short-term in order to refinance maturing debt. On the other side, the solvency is a medium to long-term concept which requires one government to fulfil its intertemporal budget constraint. Technically, this means that fiscal policy is sustainable in the long term if the present value of future primary balances is equal to the current level of debt, that is, if the intertemporal government budget constraint is met. Therefore, in a stochastic infinite-horizon economy the discounted present value of debt (in the very long term or in the infinite horizon) must be zero or negative which prevents governments from playing Ponzi Scheme (*i.e.* the No-Ponzi Game condition). Necessary conditions for the budget constraint to converge are: i) an upper bound on the primary budget surpluses-to-GDP ratio, *i.e.* fiscal limit, and ii) and a positive interest rate differential in the infinite horizon (EC, 2019).

Some of the most well-known elements in the toolkits for fiscal sustainability analysis are the fiscal reaction function and the debt accumulation equation. On one hand, the fiscal reaction function captures the reaction of the fiscal policy in terms of the budget balance to public debt and macroeconomic conditions. Bohn (1998) shows that a sustainability test relies on the estimation of a fiscal reaction function to determine whether rising debt-to-GDP ratio causes an increase in the primary budget balance. Under a stricter sustainability condition, like a debt-stabilizing policy rule, in his prominent paper Bohn (2007) proves that an upper bound on primary budget surpluses requires a stationary debt ratio for fiscal sustainability to hold. Further extensive research on the upper bound of primary surpluses has been recently explored by Davig et al. (2011), Bi and Traum (2012), Daniel and Shiamptanis (2013). Another approach to analyze fiscal policies that have gained popularity in recent years consists of the estimation of Markov regime switches to derive an endogenous and stochastic fiscal limit (Bi (2012), Bi and Leeper (2013), among others). This strand of literature interprets fiscal sustainability as a sovereign default probability, which is computed from the fiscal limit distribution. Among the few papers that address a regime-switching (or time-varying) fiscal policy rule while also, proposing a testing framework for long-run sustainability, is the one of Aldama and Creel (2020). The authors build on Bohn's (1998) Model-Based Sustainability framework and on the literature on Markov-switching fiscal policy rules that stochastically switches between sustainable and unsustainable regimes. The fiscal rule in this study describes the reaction of primary balance to the initial level of public debt. The authors define unsustainable regimes as periods with periodically and persistent explosive public debt-to-GDP ratio, *i.e.* public debt is non-stationary. Using data for France, the authors demonstrate how fiscal regimes matter for global fiscal sustainability. Aldama and Creel (2020) conclude that for the case of France, fiscal policy can be locally unsustainable, with periodically explosive (*i.e.* non-stationary) debt-to-GDP ratio and still be globally sustainable.

On the other hand, quite often the conventional debt sustainability analysis relies on the standard debt accumulation equation in order to understand the effects of macroeconomic and fiscal shocks on the evolution of debt-to-GDP ratio:

$$d_t = \frac{(1+i_t)}{(1+g_t)(1+\pi_t)}\, d_{t-1} - pb_t + dda_t \qquad (1)$$

where i_t is the nominal interest rate on government debt and g_t is the real growth rate, pb_t is the primary budget balance, π_t is the inflation rate. At given time t public debt-to-GDP ratio, d_t, is the result from interest burden on past debt, the economic growth rate and the current primary balance. The last term in (1) represents the deficit-debt adjustment *(dda),* which reconciles two key government indicators – government deficit/surplus and government debt[1]. Normally, in the empirical literature it is assumed that the size of *dda* is zero. In the absence of *dda,* the debt-to-GDP ratio evolves in line with the debt accumulation equation (1) where a surplus/deficit in a specific time will give rise to a corresponding decrease/increase in the debt level. However, the success of the consolidation in reducing debt-to-GDP ratio largely depends on the first-

[1] DDA ensures that these statistics are plausible and reliable. It explores the consistency between governments' non-financial accounts (measuring the government deficit/surplus) and financial accounts (measuring government debt) and takes into account valuation differences between the financial accounts and government debt (Maastricht debt), ECB (2018).

year value of the fiscal multiplier, which measures the impact of consolidation on economic growth, and on the reaction of sovereign yields to such a consolidation. Despite the growing research conducted in this area, there is no conclusive evidence about the size of the multipliers where estimates can span over a broad range of values. This is due to the different type of models applied to assess the multipliers, the choice of the econometric technique, the period of the analysis and other factors influencing the interest rates, the composition of the fiscal consolidation, the monetary policy response to the fiscal shock and other institutional factors such as the exchange rate. Generally, the studies of the dynamic response of macroeconomic variables to shifts in fiscal policy are focused on linear dynamics and are carried out within a Vector Auto Regression framework (VAR) and linearized (or close-to-linear) dynamic stochastic general equilibrium (DSGE) models[2]. Several studies also distinguish between fiscal multipliers in recessions and expansions, among others Kirchner et al. (2010), Baum and Koester (2011) for Germany, Batini et al. (2012) for the euro area aggregate, France, Italy, the United States and Japan, and Baum et al. (2012) for the G7 economies except Italy. Most of the studies find larger spending multipliers in recessions compared with expansions. The different size of the fiscal multiplier over time is particularly relevant for the catching-up economies of the EU, such as Bulgaria, where a number of structural changes have taken place, which undoubtedly influenced the output effects of fiscal policy. Due to these episodes of structural reforms and the observed short time-series, the estimation of the effects of fiscal policy on various macroeconomic variables, which includes the quantification of the fiscal multipliers, represents a challenge in these countries. Nevertheless, the few existing studies, mostly based on panel data approaches, suggest that fiscal multipliers in the new Member States of the EU[3]are very small or considerably smaller as compared to the estimates for the large economies, such as the USA, Germany, France and UK, mainly due to the high degree of economic openness in the smaller countries. Muir and Weber (2013) estimate the first-year spending multipliers in Bulgaria are around zero, while the first-year revenue multipliers are in the range of 0.3–0.4. The authors further find that fiscal multipliers are slightly higher in recession times then in expansion episodes. In line with most of the studies on the catching-up EU Member States, Karagyozova-Markova et al. (2013) find that for Bulgaria the effectiveness of fiscal policy in stimulating economic activity is generally low

[2] Warmedinger Th. et al (2015) provide a more detailed literature review on the topic of fiscal multipliers.

[3] Ilzetzki et al. (2010) conclude that fiscal multipliers are lower in small open economies because of the crowding out of net exports. More evidence on Central and Eastern European countries can be found in Lendvai (2007) for Hungary, Bencik (2009) for Slovakia and Mirdala (2009) for Bulgaria, Romania, Poland, Czech Republic, Slovakia and Hungary.

as first-year spending multipliers do not exceed 0.4. Employing a time-varying parameter VAR model the authors argue that in the period 1999–2007 the size of the first-year spending multiplier gradually declined, while in two years after the start of the global financial crisis the size of the multiplier doubled, before decreasing again back to its pre-crisis levels, along with the economic recovery period (2010–2011). These results indicate that the economic conditions appear to be relevant factor for the non-linear effect of fiscal policy on economic growth in Bulgaria, although the authors recognize some further research is needed to support this view.

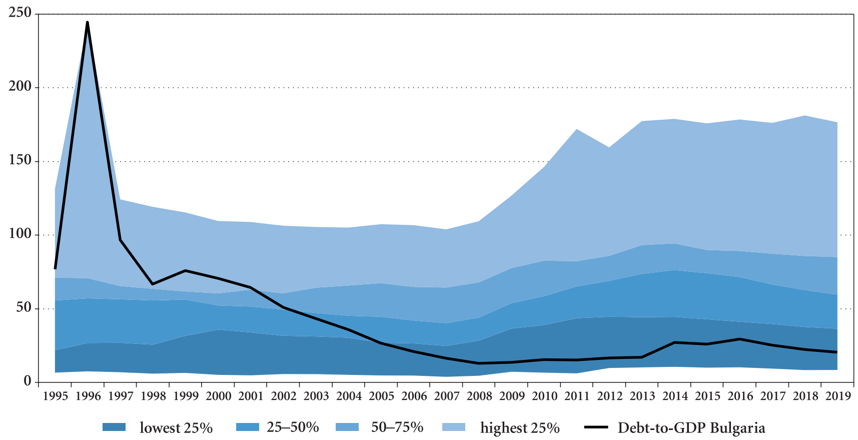
The variables usually included in the above-cited VAR studies, which analyze the effects of fiscal policy, are government spending, taxes, output and other macroeconomic variables, such as interest rates, consumption and inflation. However, the debt-to-GDP ratio is never included in these VARs, which, as pointed out by Favero and Giavazzi (2007), represents an important weakness of standard VAR models that do not account for the possible respond of government budget balance (taxes and spending) to the evolution of public debt. The authors argue that neglecting a feedback from the debt level to changes in other fiscal variable, combined with the failure to keep track of the debt dynamics, has two serious consequences. First, the estimated VAR coefficients are biased. The identified structural shocks still include the responses of the endogenous variables (such as tax revenue, government spending, interest rate) to the level of the debt ratio along the path induced by the fiscal shock. Second, due to this bias, impulse responses are sometimes computed along unstable debt paths and the interpretation of the results of usual fiscal VARs could be invalid. On the other side, including debt among the VAR's endogenous variables (Hasko, 2007 and Corsetti, Corsetti, Meier, and Muller, 2009) does not take into account the non-linear lag dynamics of the debt presented in equation (1) and may lead to miss-specification. Burriel et al. (2009), Cherif and Hasanov (2012), Tagkalakis (2014) and Attinasi and Metelli (2016), among others, follow the approach of Favero and Giavazzi (2007) and incorporate public debt in a VAR specification. Yet, very few studies track the effects of fiscal and macroeconomic shocks on public debt and the ones that do (*e.g.* Cherif and Hasanov (2012) and Attinasi and Metelli (2016)) focus on developed economies, such as the USA or selected euro area countries. To my knowledge, following the approach of Favero and Giavazzi (2007), this is the first paper to apply a debt-augmented VAR approach to a catching-up European economy in order to analyze the effects of macroeconomic and fiscal shocks on debt level, allowing for a direct response of primary budget balance and the cost of debt service to the level of the public debt. Furthermore, the explicit reconstruction of the debt trajectory, following a fiscal shock, is another important contribution of the paper to the fiscal VAR literature.

3. Structural Aspects of the Bulgarian Economy

3.1. Structure of General Government Debt

Given the historically high levels of public debt in several EU member countries, today Bulgaria stands out as one of the countries with the lowest ratios of government debt to GDP. Over the last decade Bulgarian public debt-to-GDP remains well below the Maastricht criterion of 60% of GDP, where in 2019 the debt reached a level of 20.4% of GDP (Chart 1).

Chart 1. **General Government Debt Dynamics**



Note: The EU countries are divided into four groups (quartiles) according to country's debt-to-GDP distribution for the given year.

Source: ECB.

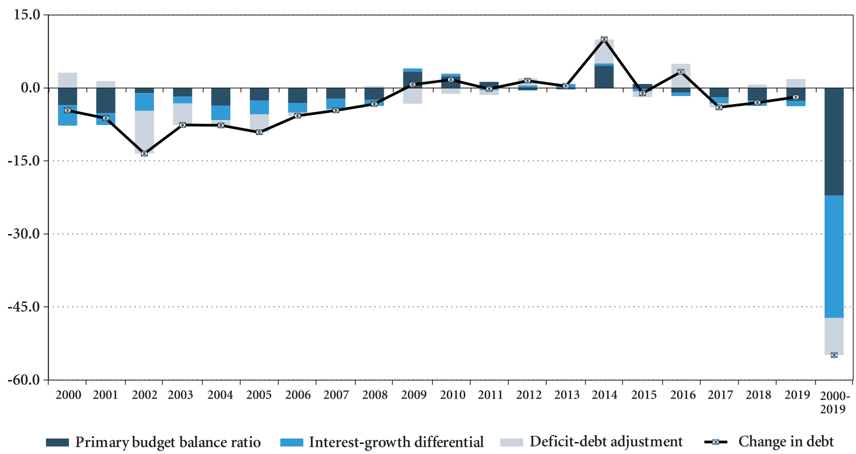
However, over the years, public debt-to-GDP ratio in Bulgaria goes through various phases, and the ratio has not always been so low. In 1996 public debt peaks at almost 250% of GDP, negatively affected by the ongoing economic reforms, the deteriorating fiscal balances and the depreciation of the lev. Following the introduction of a currency board[4] and a comprehensive stabilization package in July 1997, the country's fiscal position improves dramatically. In the following years debt remains on a downward path, reaching its lowest value of 13% of GDP in 2008. Looking closely at the factors underlying

[4] See Nenova et al (2019) for a detailed overview of the currency board arrangement in Bulgaria.

this sizable decline in public debt (Chart 2), occur in the period between 2002 and 2005, reflecting to a large extent the effects of debt restructuring, debt buyback and prepayment. Another major factor for the favorable developments in government debt in that period are positive developments in the interest-growth rate (*i.e.* $i < g$) differential that is followed by the recorded uninterrupted episodes of primary surplus between 2004 and 2008. In the years after 2009 the public debt starts to increase on the back of primary deficits and unfavorable interest-growth differentials (*i.e.* $i > g$), while deficit-debt adjustments limit the rise in the debt ratio as the government reduces the size of its financial assets. The sizable increase in debt in 2014 to 27.1% of GDP is mainly attributed to the financing of the budget deficit, the temporary accumulation of reserve, to the provided loan to Deposit Insurance Fund and the provision of liquidity to the banking sector (ECB, 2018). From 2015 onwards public debt is on a decreasing path mainly supported by favorable developments in the primary balance and interest-growth differentials (*i.e.* $i < g$). Notable exception is 2016 where the temporary peak of public debt at 29.3% of GDP is mainly the result of pre-financing operations, reflected as positive deficit-debt adjustment.

Chart 2. **Annual Changes in the Bulgarian's Debt-to-GDP Ratio and Factors behind These Changes**

(percentage points)



Notes: Factors behind changes in general government debt-to-GDP ratio (percentage points). A negative value indicates a contribution of the respective factor to a decrease in the debt-to-GDP ratio, while a positive value indicates a contribution to its increase. The snowball effect represents the interest-growth differential.

Sources: Eurostat, own calculations.

From Chart 2 it becomes clear that the interest-growth ($i - g$) differential, or so called snowball effect, is an important determinant of government debt dynamics through the period under consideration. In most of the years Bulgaria experiences a negative differential ($i < g$), which points to higher economic growth compared to the average implicit interest rate on government debt. In the period 2000–2008 the differential hoovers around -2.5 percentage points on average. These developments in the differential are supported by the strong output growth[5], observed in the period 2002–2008, which is driven by robust domestic demand, in particular buoyant investment activity, while the contribution of net exports is strongly negative. The differential turns positive (*i.e.* $i > g$) in the period 2009–2014 (with the notable exception of 2011) as the impact of the global financial and economic crisis reaches the Bulgarian economy in 2009, when the country enters into a recession. In the following years (2010–2013) the economic recovery is weak, with exports rebounding, but domestic demand being sluggish. The economy accelerates in 2014 and continues to grow steadily in the period 2015–2019 supporting the negative values of the interest – growth differentials (*i.e.* $i < g$) prevailing at that time. The drivers of growth in that period shift from the external sector to domestic demand. The increase in investments in the period 2014–2015 and 2017–2019, including those related to EU funding, likewise plays an important role in the positive developments in the economic activity.[6]

The analysis on government public debt sustainability is centered around country's debt-to-GDP ratio and its trajectory. However, the analysis should be considered along other indicators for assessing the current state of the government finances and future sustainability issues. The structure of government debt is an important source of information related to public debt sustainability. Therefore, debt composition should be analyzed along several dimensions.

With a long-lasting currency board in place which cannot finance spending of the domestic government and without independent monetary policy, the level and structure of public debt allows Bulgaria to manage its debt effectively. The share of government debt with short-term maturity (Table 1) is viewed

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[5] The *ex post* evaluation of the 'fifth wave' of EU enlargement (EUROPEAID (2015)), where among other countries Bulgaria becomes beneficiary of the PHARE [EU pre-accession] programme, estimates that the Central and Eastern Europe GDP growth was on average 4% annually in the period 1994–2008 and that the accession process itself contributed almost half to this growth.

[6] The *ex post* evaluation of Cohesion Policy programmes 2007–2013 (EC, 2016), focusing on the European Regional Development Fund and the Cohesion Fund, states that overall the support amounted to EUR 5.4 billion, equivalent to just over 2% of GDP, is estimated to have increased GDP in 2015 by almost 4% above the level it would have been in the absence of the funding provided, while GDP in 2023 will be an estimated 3% higher as a result of the investment concerned.

as a major indicator capturing refinancing and rollover risk. For Bulgaria, this indicator has generally been very low. Its share in government debt decreases form 3.9% in the period 1995–2000 to 0.1% in the end of 2019.

Table 1. **Structure of General Government Debt**

(per cent)

	1995–2000*	2000–2010*	2011	2012	2013	2014	2015	2016	2017	2018	2019
Government Debt-to-GDP Ratio	105.2	33.7	15.2	16.7	17.1	27.1	26.0	29.3	25.3	22.3	20.4
in currency other than national (per cent of total)	90.0	84.4	74.6	79.0	73.4	80.9	79.0	81.0	78.4	81.7	81.0
of which in Euro	*6.2*	*39.7*	*55.0*	*62.9*	*59.4*	*71.5*	*77.4*	*79.8*	*77.3*	*80.7*	*80.1*
Domestic ownership (per cent of total)	25.8	35.5	53.6	48.9	52.3	47.5	52.6	52.0	56.5	55.6	56.1
Government securities (per cent of total)	63.3	61.5	60.1	65.4	60.5	61.1	74.6	75.3	73.8	73.3	74.4
Short-term debt <1 year original maturity (per cent of total)	3.9	0.6	2.8	0.1	1.9	22.8	0.9	0.3	0.1	0.0	0.1
Medium- and long-term maturity debt with a variable interest rate	65.8	36.8	21.8	18.3	12.2	7.1	6.0	8.2	4.0	3.9	3.0

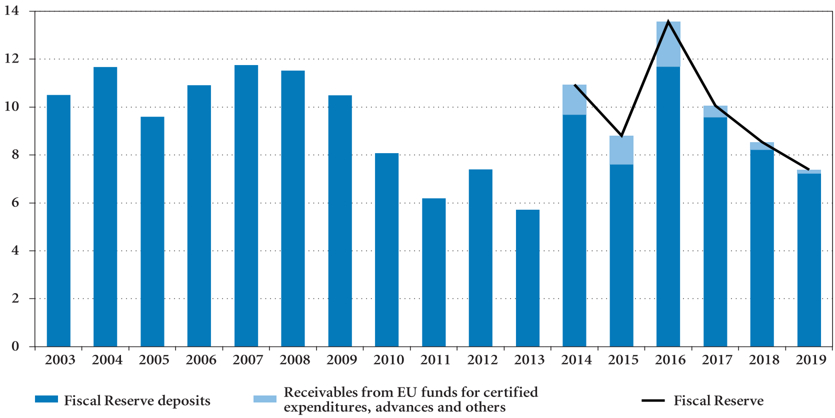
* Multi-annual averages calculated using the arithmetic mean.

Source: ECB.

Although the level of short-term indebtedness is negligible and therefore the country is not exposed to refinancing and rollover risk, from a precautionary standpoint it is worth considering the analysis of short-term debt in parallel with the availability of other liquid financial assets such as cash deposits. Chart 3 presents recent developments in Bulgarian fiscal reserve and government's cash deposits[7].

Chart 3. **Developments in the Fiscal Reserve Account**

(per cent of GDP)



Sources: Ministry of Finance, own calculations.

[7] Since 2014 the scope of the fiscal reserve has been broadened and now it includes along the Fiscal Reserve deposits the Receivables from EU funds for certified expenditures, advances and others.

The fiscal reserve account was established to facilitate the short-term financing of the budget. The accumulated funds in the fiscal reserve are held in the Bulgarian National Bank's Issue Department and are fully covered by the foreign reserves. Essentially, these are the funds, which are available to the government at any given time. However, the annual state budget law determines the minimum level of the fiscal reserve at the end of the year which provides assurance of the government's ability to honor its obligations and therefore is an important stabilizer. Given the positive developments in the fiscal reserve balance in the past 16 years (Chart 3) Bulgaria has also complied with the Greenspan-Guidotti rule of full coverage of total short-term external debt. This rule is a common practice in determining the appropriate level of cash deposits to counter a sudden stop of capital flows and country's capacity to serve its short-term debt.

Another useful indicator in analysing vulnerabilities stemming from public debt sustainability is the share of debt denominated in foreign currency that captures country's exposure to exchange rate fluctuations. The latter could increase the interest burden and affect negatively the budget balance. Yet, the proportion of foreign currency-denominated government debt has remained high in the period 2000–2019 (Table 1), reaching values above 70% of total debt. Nevertheless, foreign debt is entirely denominated in euro, which is the anchor currency of the Bulgarian currency board arrangement. The well-established currency board regime significantly reduces exposure to fiscal risks stemming from the share of public debt denominated in foreign currency. Thus, under the fixed exchange rate of the lev to the euro and thanking into consideration the low share of debt with a variable interest rates (3% in 2019), leaves the budget balance relatively insensitive to changes in interest rates.

Another important debt indicator to be taken into consideration is the share of debt held by non-residents. On one hand, high dependence on non-resident investors could be a potential concern and a source of vulnerability due to possible portfolio rebalancing in times of financial stress or heightened uncertainty, triggering substantial outflows. On the other hand, a large share of non-resident investors could undermine the credibility of a country's fiscal plans and support market confidence, thus contributing to lower debt financing. In the past few years, domestic investors have become more important holder of government debt in Bulgaria, acquiring significant share of public debt (56.1% in 2019, Table 1).

3.2. Main Developments in the Fiscal Position

After carefully reviewing debt developments and its driving forces in the recent years, the dynamics of another key indicator from the debt accounting equation (1) that influences the public debt trajectory, namely the budget balance, should also be considered.

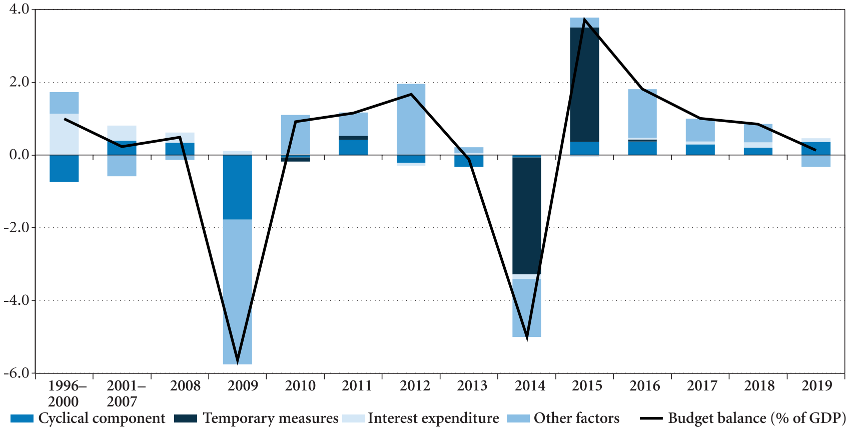
The growing debt burden until 1996, negatively affected by the ongoing economic reforms, significantly worsened the fiscal position, with interest expenditures steadily increasing until 1997. As a result, in the period 1995–1996[8] the budget balance deteriorated. Following the introduction of a currency board and a comprehensive stabilization package in July 1997, the country's fiscal position improves dramatically. With the introduction of the currency board arrangement, progress is made in macroeconomic stabilization and inflationary financing of the budget deficit by the central bank is discontinued. In the period 1997–2008 the fiscal budget is on surplus with the notable exception of the years 2000 and 2002. Both structural and cyclical factors drive the improvement in the fiscal position (ECB, 2008)[9]. However, with the onset of the global crisis in 2009, Bulgaria sees a significant deterioration in its budgetary position. In the period 2009–2015, the general government budget balance is in a continuous deficit, breaching the reference Maastricht value of 3% only in 2014. Some of the factors contributing to the worsening of the budget balance through these years are the one-off capital transfer (amounting to 3.1% of GDP) related to the reclassification of the Deposit Insurance Fund within the government sector, sizeable revenue shortfalls and a large increase in public investment (ECB, 2018). In the period 2016–2019, the prudent fiscal policy implemented by the government leads to a significant improvement in the budget position and to the recording of fiscal surpluses in four consecutive years. The consolidation process in this period is determined by structural and cyclical factors (Chart 4).

[8] 1995 is the first year for which data on government financial statistics are available in Eurostat database.

[9] Some of the major legislative changes that take place in the period 2000–2019 are: a significant reform in the area of social security implemented in 2000 which includes: introduction of a new pension formula, increase of the pension age, establishment of a differentiation by funds of the socially insured risks, introduction of the second pillar with the universal pension funds by diverting a specified share of insurance contributions from the first pillar and streaming it to the second one. Further to these reforms, the following legislative changes take place: a reduction in the corporate tax rate from 19% to 15% in 2005 and further to 10% in 2007; transition from a progressive tax rate to flat tax rate on personal income tax (2008); reduction of social security rate by 3 percentage points and adjustment in the proportion of social security contribution paid by the employer and the employee (2008); gradual increase through the years in minimum and maximum insurable income; increase in the excise rates through the years; reduction of the total social contribution by 2 percentage points (2010); increase of pension contributions with 1 percentage point (2017).

Chart 4. **Factors behind Changes in Government Budget Balance-to-GDP Ratio**

(percentage points)



Note: The chart presents the decomposition of the changes in the nominal government budget balance in the period 1996–2019.

Sources: Eurostat, own calculations.

4. Empirical Model and Estimation

4.1. Empirical Model

In this paper the approach of Favero and Giavazzi (2007) is followed in order to study how shifts in fiscal policy and major macroeconomic variables affect debt dynamics and primary balance, GDP growth, inflation, average implicit interest rate[10] on government debt. The endogenous macroeconomic variables are allowed to respond to the level of debt-to-GDP ratio, included as an exogenous variable, which evolution over time is determined by the intertemporal budget constraint. The model specification followed in this study is:

$$Y_t = \sum_{i=1}^{k} \Phi_i Y_{t-i} + \sum_{i=1}^{l} \gamma_i d_{t-i} + \varepsilon_t \qquad (2),$$

$$d_t = \frac{(1+i_t)}{(1+\pi_t)(1+g_t)} d_{t-1} + pdef_t \qquad (3).$$

The endogenous vector Y specified in (2) consists of four variables [$pdef_t$, g_t, i_t, π_t], where $pdef_t$ and g_t are, respectively, the primary deficit-to-GDP ratio

[10] The average nominal implicit interest rate, i_t, is obtained by dividing interest payments at time t by total stock of general government debt at time $t-1$.

(primary expenditure minus revenue) and real GDP growth rate, i_t average nominal implicit interest rate on outstanding debt and π_t is inflation rate based on the GDP deflator. d_t is the debt-to-GDP ratio. K and l represent the number of lags chosen for the endogenous and the exogenous variables ($k = 4$, $l = 2$) and Φ_i is the time invariant matrix of coefficients and Y_i is the coefficient to the exogenous variable. As in Favero and Giavazzi (2007) the current analysis incorporates a lag length of four quarters for the endogenous variables in order to ensure serially uncorrelated residuals[11]. The inclusion of four lags is a common practice in the estimation of models with quarterly data (Caldara and Kamps, 2008). Furthermore, it is unlikely, that short lags of the variables $pdef_t$, g_t, i_t, π_t would trace the evolution of debt ratio accordingly. The current study incorporates a lag length of two for the debt-to-GDP ratio (see Appendix A for the conducted tests). Equation (3) describes debt dynamics and is needed to compute impulse responses. It represents a deterministic equation necessary to reproduce debt series from the endogenous variables in equation (2).

4.2. Model Estimation and Impulse Response Functions

The VAR is estimated *via* OLS. Equation (3) is an identity with no estimation parameters, thus does not require any estimation. In the computation of forecasts or impulse responses, debt at each period is calculated based on the predicted macroeconomic variables from (2) and then inserted back into the government intertemporal budget constraint (3). Including the debt ratio as an exogenous variable in equation (2) keeps track record of the debt feedback and thus computation of the traditional orthogonalized impulse responses (OIR) might be difficult. To take into account the nonlinear relationship among the variables imposed through the inclusion of debt-to-GDP ratio in the VAR, the estimated impulse responses in this study represent generalized impulse response (GIR) functions as proposed in Koop, Pesaran and Potter (1996) for non-linear models and discussed in further details in Pesaran and Shin (1998). The GIR functions are an alternative to the OIR functions of Sims (1980). See Appendix B for a detailed description of the GIR and OIR.

GIR functions show what has happened historically when, for example, primary balance moves first, that is, when primary balance shocks have occurred before other shocks and will be accompanied with shocks to economic growth, inflation, and interest rate as observed empirically. In this particular order, the GIR's formulation of a primary deficit shock would also be identical to a shock derived from Cholesky identification. The GIRs provide an opportunity to

[11] Formal tests as the Akaike information criterion (AIC) and other the Schwarz Criterion (SC) performed on the Bulgarian data suggest the inclusion of one lag for the endogenous variables.

examine out-of-sample forecasts and impulse responses. The computed IRs are defined as the difference between the two expectations conditional on an initial shock (ϑ_t) and history (k_{t-1}). k_{t-1} is a specific realisation of the information set $K_{t-1} = (y_{t-1}, y_{t-2}, \ldots)$ for the response and on the history (k_{t-1}) for the baseline:

$$GIR(Y;\ \vartheta_t, \kappa_{t-1}, h\ ) \ = \ E[Y_{t+h}|\varepsilon_{jt} = \vartheta_{jt}, \kappa_{t-1}] \ - \ E[Y_{t+h}|\ \kappa_{t-1}] \ \ for\ h \ = \ 0, 1, 2 \ldots \tag{4}$$

In particular, the response at horizon h of a variable Y to a shock at time t (ϑ_t) is given by the difference between the expected value of variable Y given the shock (ϑ_t) and conditional on a particular history (k_{t-1}) of the shocks at time $t-1$ and the expected value of Y in the absence of such a shock. Unlike the OIR, the GIR functions do not require the researcher to make assumptions about the ordering of the endogenous variables in the model, which is clearly an important consideration given various possible alternative orderings.

Koop, Pesaran and Potter (1996) argue that OIR is more usefully applied to linear models than to non-linear ones. The basic object that is required in the computation of GIR is conditional expectation (see Appendix C for the various methods of computing the conditional expectation presented by Granger and Teräsvirta (1993)). The following text describes the steps involved in computing the conditional expectations in the GIR in the current study. The impulse responses are computed *via* a bootstrapping procedure as described in details in Koop, Pesaran and Potter (1996). More specifically, the following procedure has been followed:

1) The VAR in (2) is estimated *via* OLS. The variance-covariance matrix for the innovations $(\hat{\Omega})$ is calculated. The estimated residuals are transformed to obtain a contemporaneous independence by using the inverse of a Cholesky factorization of the estimated covariance matrix ($\hat{u}_t = P^{-1}\hat{\varepsilon}_t$, where P is the lower triangular Cholesky decomposition of $\hat{\Omega}$, such that $\hat{\Omega} = PP'$). This creates a sample of size NxT with the elements from the $\hat{u}_t$ to bootstrap from. For the j^{th} replication a bootstrap procedure[12] with replacement is utilized to draw an unordered collection of NxT[13] items. By assumption, the elements of the latter collection are independently distributed both contemporaneously and serially over time. From this collection, denoted by (u_t^j) is recovered $\varepsilon_t^j = Pu_t^j$;

[12] Here the built-in Matlab function 'bootstrap' was utilized. The bootstrap procedure involves choosing random samples with replacement from a data set and analyzing each sample the same way. More specifically, resampling means that each observation is selected separately at random from the original dataset. The number of elements in each bootstrap sample equals the number of elements in the original data set. Matlab 2020b, 'Statistics and Machine Learning Toolbox'.

[13] Where the number of endogenous variables is N and the estimation sample is T.

2) Use the VAR model in (2) and the shocks generated in step 1) to compute simulated realizations of Y and corresponding d;

3) For a given horizon H, randomly sample (H+1)xR values of the (4-dimensional) innovation, where R is the number of replications (R=1000);

4) Useing the first H random shocks (obtained under step 3) to compute the realizations $y^0_{t+h}(\vartheta_t, \kappa_{t-1})$ for h = 0,1,2...H iterating on the non-linear time VAR given the initial conditions $\vartheta_t, \kappa_{t-1}$ ('shocked' variable). For each realisation y^0_{t+h} compute shocked debt-to-GDP ratio $d^0_{t+h}(\vartheta_t, \kappa_{t-1})$ *via* (3);

5) Compute the realizations $y^0_{t+h}(\kappa_{t-1})$ for h = 0,1,2,...H ('no shock' variable) and the corresponding debt-to-GDP ratio $d^0_{t+h}(\kappa_{t-1})$;

6) Repeat steps 4 and 5 R times and calculate the average for each individual component:

$$\bar{y}_{R,t+h}(\vartheta_t, \kappa_{t-1}) = \frac{1}{R}\sum_{i=0}^{R-1} y^i_{t+h}(\vartheta_t, \kappa_{t-1}), \text{ h} = 1,2,..,\text{H};$$

$$\bar{y}_{R,t+h}(\kappa_{t-1}) = \frac{1}{R}\sum_{i=0}^{R-1} y^i_{t+h}(\kappa_{t-1}), \text{ h} = 0,1,2,..,\text{H};$$

As $R \to \infty$, by the Law of large Numbers these averages across individual iterations will converge to the conditional expectations, namely $E[Y_{t+h}|\vartheta_t, \kappa_{t-1}]$ and $E[Y_{t+h}|\kappa_{t-1}]$;

7) Take the difference between the two averages to form the bootstrapped estimates of the GIRs;

8) Repeat steps 1 to 6 1000 times to allow accurate estimation of the GIR random vector representing the difference between the average 'shock' and 'no shock' forecast paths.

Following the above listed steps and averaging across history[14] (k_{t-1}) leads to the calculation of mean GIR conditional only on the shock which are later presented in Section 5. However, as pointed out by Koop, Pesaran and Potter (1996), if shocks have asymmetric effects over the business cycle, then averaging across phases of the business cycle will tend to weaken or hide the evidence of the asymmetry. Therefore, the analysis in this study of debt response under different macroeconomic and fiscal shocks relies on the estimated mean generalized impulses conditional on a specific history (k_{t-1}), where the whole historical sample from 2000:Q2 until 2019:Q4 has been employed as a particular history

[14] We draw 1000 times from the joint distribution of the innovations at each history in the sample to produce 1000 realizations of the shock for each k_t. Thus, given the 79 histories, we generate 79,000 realizations of the GIRs from which we calculate the mean GIR conditional only on the shock.

(k_{t-1}). Thus, the non-linear model (Equation 2) allows for the effects of the shocks to very across business cycle through changes in the conditional mean of the generalized debt *IRs*.

5. The Data

5.1. Data

In this paper, quarterly data of the Bulgarian economy is used over the period 2000:Q2 – 2019:Q4 (the first period for which the quarterly debt data are available in Eurostat). The data source for the budget balance, interest payments on government debt, the stock of government debt and for the macroeconomic variables, is Eurostat – government finance statistics and national account data. All data are seasonally adjusted by Eurostat.

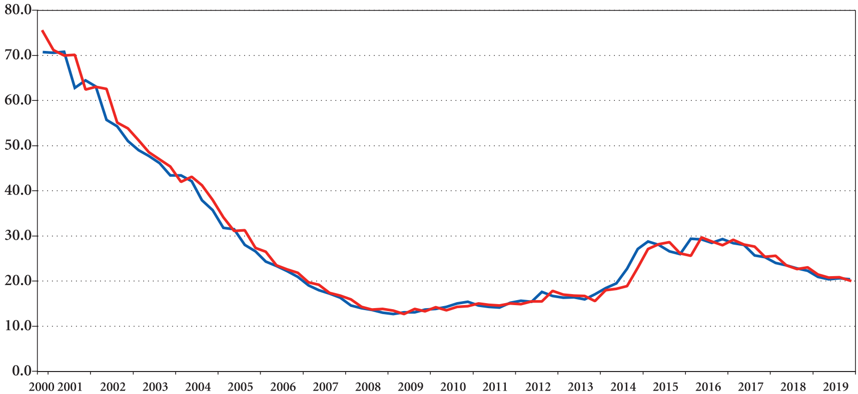
Nominal government primary-deficit-to-GDP ratio, *pdef,* represents primary government expenditure minus revenues. g_t is the quarterly growth rate of real GDP. The average nominal implicit interest rate, i_t, is obtained by dividing interest payments at time t by total stock of general government debt at time $t-1$. π_t is inflation rate, calculated as the log difference of GDP deflator. Finally, d_t is debt-to-GDP ratio at time t. In the current framework the implicit interest rate is used instead of the market interest rate in order to be able to reconstruct the debt level from the primary surplus and the other variables present in the debt accumulation equation (3).

5.2. Debt Reconstruction

In order to apply the debt augmented VAR framework (equation 3) to a country with a currency board arrangement, the accuracy of debt dynamics in (3) is checked by simulating the debt path forward from 2000:Q2 *via* the deterministic debt equation in (3). The simulated debt-to-GDP ratio for each period t is calculated using the historical time-series in period t, namely primary balance, economic growth rate, implicit interest rate, inflation rate, as well as debt-to-GDP ratio in period $t-1$, which are then replaced in the deterministic debt equation (3). Thus, the simulated debt-to-GDP ratio in period t is calculated. Chart 5 presents a comparison between the simulated debt-to-GDP ratio based on (3) and the actual value of the debt-to-GDP ratio.

Chart 5. **Bulgaria Actual Debt-to-GDP Ratio *vs* Simulated Debt-to-GDP Ratio**

(per cent of GDP, 2000:Q2–2019:Q4)



Note: The chart compares actual value of debt-to-GDP ratio (*blue line*) with the simulated (calculated based on equation (3) value of debt-to-GDP ratio (*red line*) for the period 2000–2019).

Sources: Eurostat, own calculations.

In some periods the actual and simulated debt ratios diverge (Chart 5). The main reason behind these inconsistencies is the fact that the debt accumulation equation (3) used to reconstruct the debt series typically includes a third variable which is the deficit-debt adjustment (DDA)[15]. Overall, the average discrepancy between the actual and the simulated debt series is -0.4%, while some more significant deviations (around 4.5%) are observed in the beginning of the sample 2000–2005 and later in 2014 and 2016[16]. Major role for these deviations plays the net accumulation of currency and deposits[17] by the government. Thus, as a robustness check, the simulated debt ratio is calculated following the same procedure as described above, where gross debt is replaced with net government debt (*i.e.* gross public debt minus currency and deposits for general government sector). This procedure, leads to some improvement in the fit of the simulated debt ratio for the period 2015–2019 and to some worsening of the fit in the other periods of the sample. On average, the discrepancy between the simulated and actual debt ratio remains broadly unchanged (-0.34%). However, for comparability of the analytical results in the current debt augmented VAR framework gross public debt is the preferred choice. This will also ease the

[15] The three pillars of the DDA are the three groups of factors which can contribute to the DDA – namely, (i) transactions in main financial assets, (ii) valuation effects and other changes in the volume of debt, and (iii) time of recording differences and remaining factors.

[16] Please refer to section 3 for an overview of the driving forces behind the DDA item.

[17] On average, the DDA is 1.1% of GDP in the period 2015–2019, with a significant contribution (0.6% of GDP) to the currency and deposits component (ECB, 2020).

assessment of the compliance of the projected debt path with EU numerical debt rule (Maastricht Treaty's 60% reference value). Furthermore, following the standard approach in the empirical literature it is assumed that the size of *DDA* is zero (Cherif and Hasanov (2012), Attinasi and Metelli (2016)). Thus, in the absence of *DDAs*, the debt-to-GDP ratio evolves in line with the government budget constraint (3) where a surplus/deficit in a specific time will give rise to a corresponding decrease/increase in the debt level.

6. Public Debt Dynamics and Impulse Responses

This section presents the GIRs associated with the following shocks: austerity shock (negative shock to the primary deficit), real GDP growth shock, and inflation shock. The GIRs[18] are computed as described in Section 4. In nonlinear models, responses depend on the sign and the size of the shock as well as on the initial conditions up to the point where they are computed. In the presented impulse responses the size of the shock is held constant and the impulse responses are calculated on i) a specific economic state, conditioning on each quarter from the sample 2000:Q2 until 2019:Q4, and ii) averaging the initial economic conditions throughout the whole sample period.

If the time series is stationary, then as the horizon approaches infinity the GI random vector converges to a vector of zeros. That is, under stationarity the baseline forecast, and shocked forecast converge to the same value for any history. Alternatively, if the time series under consideration are random walks, then the dispersion of the GI remains constant as the horizon is increased.

6.1. Debt Impulse Responses to an Austerity Shock

The main channels through which fiscal consolidation affects the evolution of the debt ratio are through the implication on the primary balance, GDP growth rate and the average implicit interest rate. The change in primary balance is driven directly and indirectly by the consolidation measures. The direct effects result from the fact that consolidation reduces primary deficit, while the indirect measures are given *via* the effect on growth. Blanchard and Perotti (2002) find consistent evidence that positive tax shocks have a negative effect on output where the magnitude of this effect depends on the size of the fiscal multiplier (which measures the change in output following a change in the deficit). Lower growth triggers automatic stabilizers *via* lower revenues and higher expenditure that limit the improvement in the fiscal balance. Revenues (except for non-tax revenues) tend to rise and fall in line with fluctuations in economic activity. On

[18] In each graph, the horizontal axis represents quarters and the vertical axis the size of the shock to each variable.

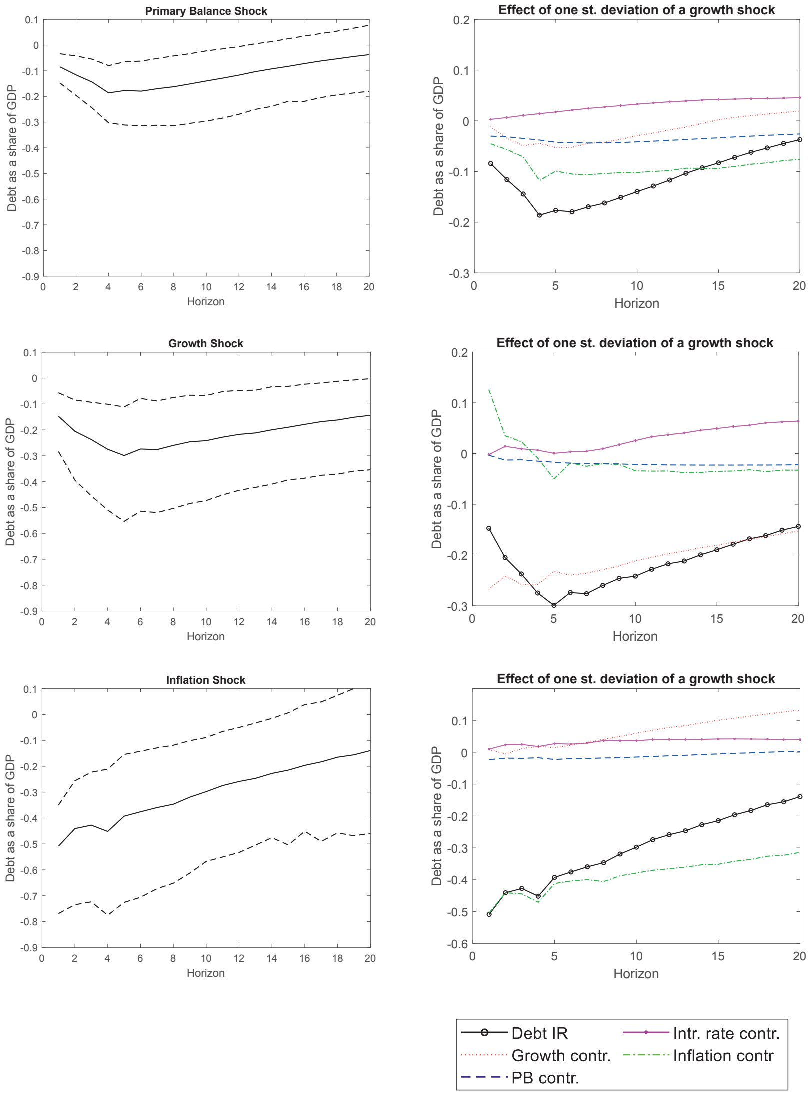
the expenditure side, unemployment benefits are the main government outflow component assumed to react 'automatically' to the cycle which for example in economic downturn drives government expenditure up. Thus, the primary balance is increased by the direct effect of consolidation measures but usually reduced by the impact that these measures have on the economic growth rate (indirect effect). As a result, the debt-to-GDP ratio does not decrease one-for-one with fiscal consolidation. On the other side, the impact of a consolidation shock on the average implicit interest rate is better visible in the medium-term than in the short term[19]. The sign of this effect however is not clear-cut as it depends crucially on the way market expectations are generated. In the normal case, fiscal consolidation measures increase market's confidence in government bonds, which translates into reduced yields.

Turning to the debt impulse responses following an austerity shock (a shock that leads to a fiscal tightening), averaging across the initial economic conditions in the period 2000:Q2 to 2019:Q4 produces impulse responses, which are only shock-dependent. This eliminates the possible danger of choosing arbitrary a particular history which inevitably results in the estimation of different debt impulse responses. In the case of an austerity shock and under average economic conditions, debt-to-GDP ratio would decrease by about 2.6% of GDP in 5 years and then returns to its pre-shock baseline path. However, the decline in debt-to-GDP ratio is statistically significant only in the first two and a half years following the shock. Chart 6 shows the debt impulse response to one standard deviation increase in the primary surplus and the contribution of the macroeconomic variables to debt dynamics where the initial economic conditions are averaged.

The average decline in debt ratio in the first few quarters is driven by higher inflation, primary surpluses and positive effects on the economic growth, whereas the implicit interest rate counteracts the fall in debt ratio (see Appendix D for the derivation of the contributions). Despite the average initial shock of one standard deviation increase in the primary surplus (0.03% of GDP) on impact, over the following 5 years, the primary balance continues to improve, increasing on average by another 0.7% of GDP thus supporting the debt decline. The short-term effects of fiscal shocks on output are captured by the fiscal multiplier. While there is a general agreement that fiscal consolidation has a negative effect on GDP in the short-run, the size of its impact depends on several factors

[19] Only a part of the share of overall debt needs to be rolled over in every one year (the average maturity of Bulgarian government debt is 6.9 years, data up to 2019), so the effect of fiscal consolidation on the average interest rate is more muted.

Chart 6. **Debt Impulse Responses to Macro Shocks and Decomposition: Average Conditions, GIR Identification**



Notes: The chart shows the median responses of the debt ratio and the 95% confidence bands to one standard deviation positive shocks in primary balance (pb), growth (y), inflation (π), and interest rate (i) under GIR identification. The decomposition of the responses to contributions from primary deficit, growth, inflation, and interest rate is also presented. The contributions do not add up exactly due to rounding.

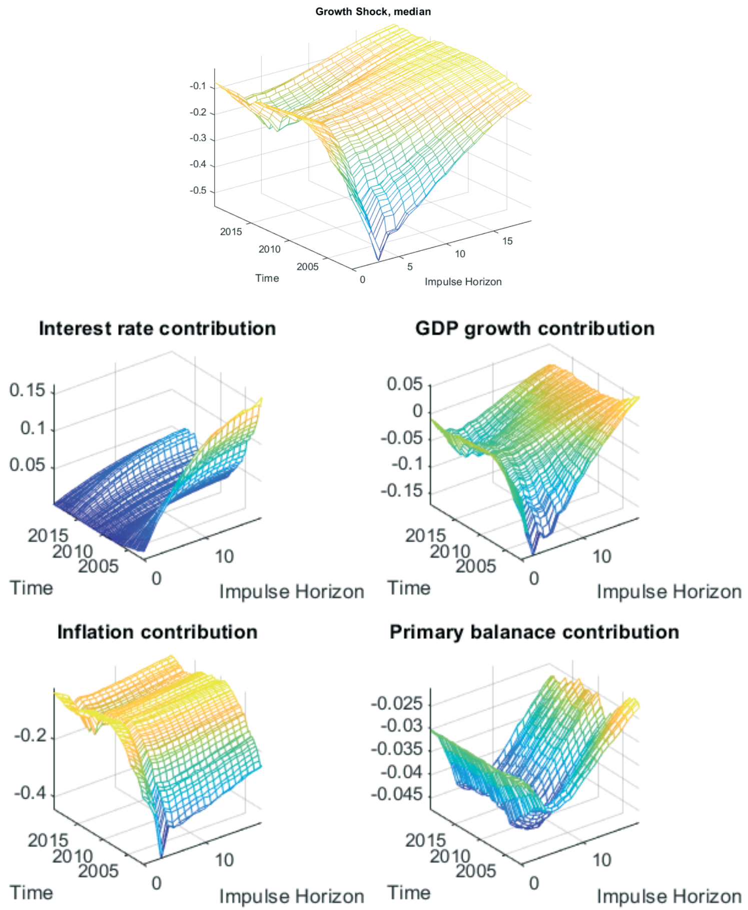
which are well described in the European Commission (2012)[20]. As a result of an austerity shock, with either average initial conditions or taking each quarter in the sample as specific economic history, growth increases but this response is statistically insignificant in the first 5 quarters. Later the GDP growth response switches to the more familiar negative territory and becomes statistically significant. Overall, the higher growth in the first few quarters contributes to lower debt ratio. The analysis of the estimated VAR coefficients suggests that the negative sign of the estimated VAR coefficient of the first lag of primary deficit variable (primary expenditure minus revenue) in the growth equation contributes to the puzzling positive growth response given an episode of fiscal consolidation in the first few quarters. The increase in economic growth following a fiscal consolidation shock could possibly be explained by the fact that announcements of tax increases tends to boost economic activity temporary, while the impact upon implementation of the reforms remains negative. One possible reasoning behind this is that, agents consume before the permanent tax hikes hits. Alternatively, announcements of consolidation measures may temporarily increase confidence in the economy. Similar results for Bulgaria are found in Mirdala (2009) and Karagyozova-Markova, Deyanov and Iliev (2013). On the other hand, averaging the impulse response functions over the whole period from 2000:Q2 to 2019:Q4, shows that the implicit interest rate increases due to an austerity shock. However, it should be noted that the interest rate responses are statistically insignificant with 95% confidence intervals[21] throughout the measurement period.

Given that the model with the debt feedback is nonlinear, the initial economic conditions may matter. With initial conditions as in the beginning of the sample the decline in debt-to-GDP ratio is more profound following an austerity shock but the debt response turns statistically insignificant after only year and a half (Chart 7).

[20] These changes are grouped as follows: i) factors that force consumers to base consumption choices on current revenues only, such as financial frictions; ii) factors concerning the nature of the fiscal shock, in particular the credibility of the shock and/or its permanent or temporary nature; iii) the composition of the fiscal shock; iv) structural features of the economy, like the presence of nominal or real rigidities; v) the size of automatic stabilizers; vi) the type of monetary policy, and vii) the exchange rate regime and the degree of openness of the economy.

[21] The confidence intervals reported in the case of average initial economic conditions represent the median of the lower and upper percentile of the GIRs conditional on each data point in the sample.

Chart 7. **Median Debt Impulse Responses to a Primary Balance Shock and Decomposition, GIR Identification**

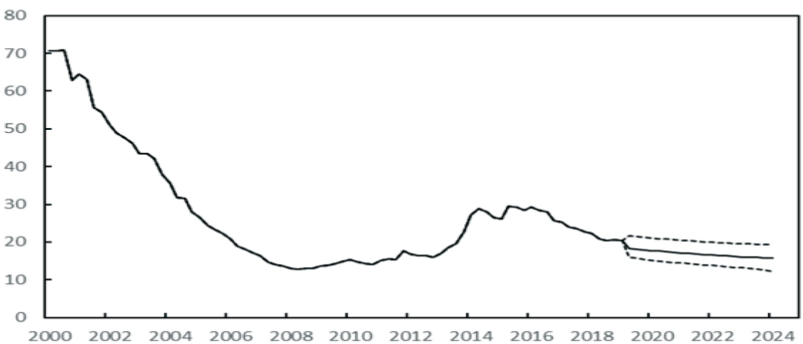


Notes: The chart shows the debt response to one standard deviation positive shock in primary deficit (pb) with initial economic conditions of each quarter throughout the sample (first chart). The decomposition of the response to contributions from primary deficit, growth, inflation, and interest rate is also presented (see Appendix D).

It is likely that the magnitude of the response of economic growth to fiscal shocks, which is related to the concept of fiscal multipliers, is not the same throughout the years, as it looks it depends on the economic situation (Chart 7) prevailing at the time of the shock. In times of more depressed economic environment and elevated public indebtedness, an austerity shock would result in a more profound increase in economic growth as compared to the one seen in times with lower level of debt. Nevertheless, this puzzling positive response of real growth is short lived and the economic growth becomes negative after few quarters. On the other hand, looking at the interest rate contribution throughout the first years of the sample, gives us information about the response of the implicit interest rate based on a specific economic history (Chart 7). The implicit interest rate reacts with some degree of variation over time. It seems that the interest rate response is stronger in times facing high public debt ratio and depressed economic environment. This is in line with Engen and Hubbard (2004) and Gale and Orszag (2004) growth theory showing that the debt level rather than the government deficit is the relevant fiscal variable that has an impact on long-term interest rates. In the years (2006–2008 and 2012–2019) when the macroeconomic and fiscal conditions stabilize and the debt level is at lower levels, with initial economic conditions as the ones after 2006, debt reduction following an austerity shock is lower. Nevertheless, the confidence bands become tighter and the debt decline is statistically significant for a longer period of about 3 years.

In summary, the magnitude of public debt fall due to an austerity shock highly depends on the specific economic conditions, and the level of public debt, prevailing at time of fiscal consolidation. In times of high public debt and depressed economic state, an austerity shock leads to a higher and more persistent debt reduction. However, the austerity shock-induced decline in debt fades away in 5 years and government debt converges to a long-run value. Driven by improving economic conditions and decreasing deficits as seen in the past, the debt ratio is projected to fall (Chart 8).

Chart 8. **Historical Value of Debt-to-GDP and Out-of-sample Forecast for the Period 2020: Q1 – 2024:Q4, Average Initial Economic Conditions**



Notes: The chart shows the debt ratio time series from 2000 until 2019 and the median 5-year forecast from the first quarter of 2020 based on an estimated VAR model with debt feedback (as described in the text), averaging the initial economic conditions. The 95% confidence interval is also presented.

6.2. Debt Impulse Responses to a Growth Shock

There are two main channels by which increases in economic growth can contribute to debt reduction. First, economic upswings positively impact debt-to-GDP ratio through the denominator effect. This effect operates through the interest-growth differential. As the differential affects debt level in every period, persistent shifts in economic growth rate are compounded and have strong impact on long term debt dynamics, especially when initial debt levels are high[22]. A second effect occurs indirectly and operates through the effect of economic growth on the government budget balance. There is a broad consensus that at least in the short run, economic upturns boost government's revenue base without generating expenditure pressures. As a result, government balance tends to improve *via* generated unexpected revenue windfalls which could be used to lower public debt[23]. The magnitude of the response of the budget balance to economic growth depends crucially on the size of automatic stabilizers *via* tax revenues, social contributions and unemployment related expenditures.

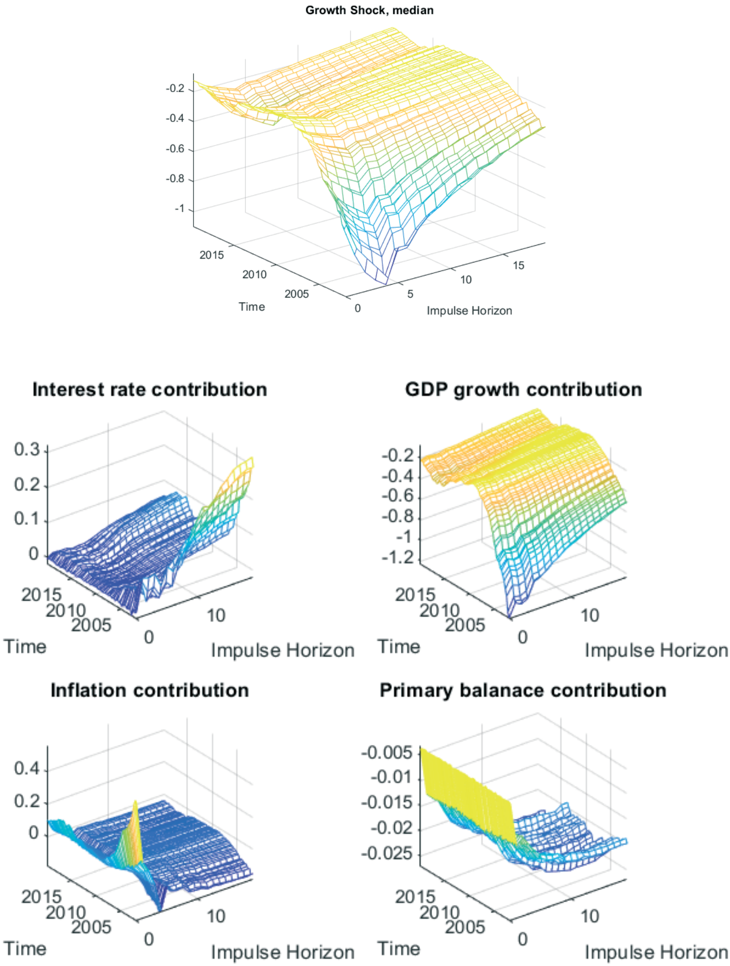
Turning to the impulse responses, following a positive shock to growth and averaging the initial conditions, debt-to-GDP ratio would unsurprisingly decline first and later converges back to its pre-shock path. The growth shock has a statistically significant effect on debt ratio throughout the whole forecasting horizon (Chart 6). With a growth shock of one standard deviation (about 0.02%) on impact, the debt ratio falls by about 1.5% of GDP in the first year. Debt reduction in the first year is supported on average by the increasing share of primary surpluses as growth continues to increase by about 1.5% *per* year in the first two years, while higher inflation rate and interest rate on government debt counteract the effect of the positive growth shock. Yet, the effect of primary balance is statistically insignificant throughout the whole forecasting horizon. Averaging the initial conditions, in 5 years, the debt ratio falls by 5.6% of GDP supported by a cumulative increase in primary surpluses, amounting to 0.4% of GDP and by lower inflation (by 0.8%) where the initial increase in the latter quickly dies out after the third quarter. The inflation response becomes statistically significant after the first two quarters. Again, due to the non-linear model, the impact of the growth shock on debt ratio depends on the initial economic conditions and the level of debt. A positive growth shock

[22] For the current discussion, it is assumed the interest rates in every period in equation (15) are unaffected by higher growth.

[23] This effect would be permanent for policies stimulating potential GDP and transitionary for cyclical shocks that do not affect GDP in the long run.

hitting the economy in times of elevated debt levels and depressed economic conditions seems to lead to a larger debt reduction (Chart 9).

Chart 9. **Median Debt Impulse Responses to a Growth Shock and Decomposition, GIR Identification**



Notes: The chart shows the debt response to one standard deviation positive growth shock with initial economic conditions of each quarter throughout the sample (first chart). The decomposition of the response to contributions from primary deficit, growth, inflation, and interest rate is also presented (see Appendix D).

6.3. Debt Impulse Responses to an Inflation Shock

Higher inflation is one of the channels through which debt reduction could be achieved[24]. A surprise increase in inflation reduces the debt level through its effect on primary balance and growth. First, for any given debt stock and real growth rate, inflation mechanically decreases the debt-to-GDP ratio. It increases nominal GDP, pushing the debt-to-GDP ratio down[25]. Second, high inflation has different effects on the nominal primary balance which work in opposite direction. The magnitude of these effects on the primary balance generally depends on respective elasticities and lags with which revenue and expenditure react to changes in the economic conditions. However, in practice it is far more complex to assess these effects much dependent also on the structure of the budget revenue and expenditure, the tax policy in place, the specific collection mechanisms. On the revenue side, direct taxes response to high inflation would largely depend on the sensitivity of the tax bases (wages and firms' profits) to such a shock, while not all indirect taxes would react to high inflation[26]. Conversely, high inflation is expected to have a negative impact on budget expenditure. Although in Bulgaria there is no mechanism for mechanical indexation of certain expense, budget outflows are expected to be negatively impacted by high inflation as a significant part of social payments are influenced by changes in price level and wage dynamics with a certain lag. Nevertheless, the effect of an unexpected shock to inflation on the government expenses could be more muted as budgets are usually prepared and executed in nominal terms, thus it may be difficult to adjust spending lines to unexpected deviations from the budget forecasts within a given fiscal year. Thus, the higher inflation has a variety of effects on the public finances and its overall effect depends on the sensitivity of individual fiscal items on changes in inflation rate.

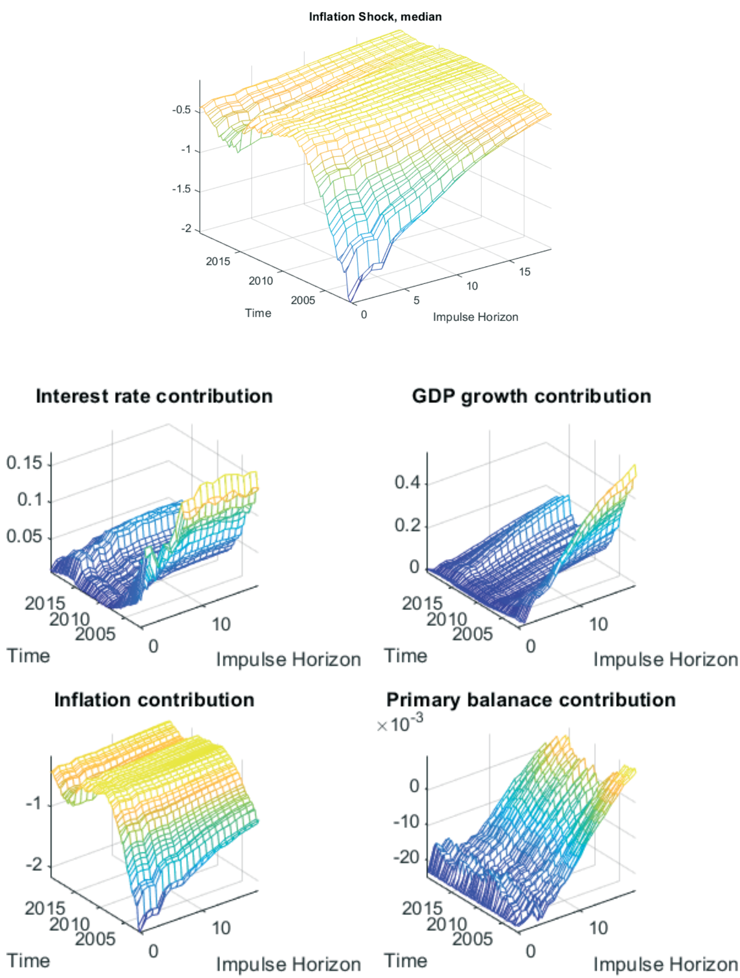
Returning to the impulse responses, if monetary and fiscal policy react to higher inflation as observed in the past, averaging the initial economic conditions, one

[24] Best et al. (2019) describe a broad set of policy options for public debt reduction which include growth-enhancing policies, fiscal consolidation, monetary policy, and financial repressions.

[25] Another channel through which higher inflation might affect the evolution of debt ratio is the interest rate on new and rolled over debt. The implicit interest rate on government debt could raise after a surprise increase in inflation, which will cause the debt ratio to grow. However, the possible effect of higher inflation on the interest rate would be better seen in the medium-term than in the short-term as each year only part of the total debt needs to be repaid through the issuance of new debt. The above derivation of the model, however, abstracts from this effect.

[26] For example, excise taxes, fees, fines, *etc.*, where the payable amount is calculated in relation to the number of transactions rather than their value, the nominal tax liability remains unaffected by inflation.

Chart 10. **Median Debt Impulse Responses to an Inflation Shock and Decomposition, GIR Identification**



Notes: The chart shows the debt response to one standard deviation positive inflation shock with initial economic conditions of each quarter throughout the sample (first chart). The decomposition of the response to contributions from primary deficit, growth, inflation, and interest rate is also presented (see Appendix D).

standard deviation inflation shock (about 0.035% of GDP) on impact, results first in a decrease in debt-to-GDP ratio of around 0.5% of GDP on impact (Chart 6). The decline in debt ratio is statistically significant for the first three and a half years. Nonetheless, the uncertainty around the median debt ratio path is higher in the current shock suggesting debt reduction could be more gradual. In the medium term under an inflation shock debt-to-GDP ratio declines by 7.5 percentage points but gradually reverts to its pre-shock level at the end of the forecasting horizon. Following the shock, the change in debt is driven by a combination of opposing forces – debt decline is supported by higher inflation and improvements in the budgetary balance throughout the next 5 years, whereas the increase in borrowing costs translated in higher implicit interest rate counteracts the fall in debt ratio.

Given the non-linearity of the model, the magnitude of debt decline depends on the initial conditions prevailing at the time of the shock. In times when the country is confronted with elevated debt close to the one witnessed around the period 2000–2004 accompanied with higher implicit interest rate on debt an inflation shock would lead to a sharp decline in the debt-to-GDP ratio in the first year and gradually to return to its pre-crisis level (Chart 10).

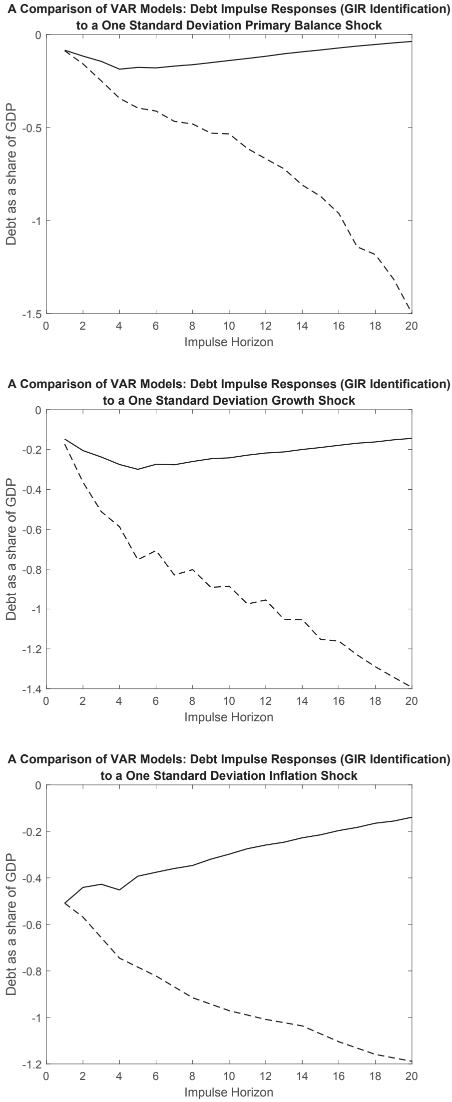
Nonetheless, the uncertainty around the median debt ratio path is higher in the current shock suggesting debt reduction could be more gradual.

7. Sensitivity Analysis

This section presents some robustness exercises carried out along two main dimensions. First, the sensitivity of the results is assessed with respect to structure of the model where the results from a VAR model with debt but without debt feedback are analyzed[27]. A VAR model that excludes debt feedback could imply an unrealistic (explosive) debt path. Chart 8 reports the results for the specification without debt feedback. Averaging the initial economic conditions, and eliminating the debt feedback in the VAR equation (2), the VAR results in explosive debt dynamics (Chart 11) and persistent impulse responses of the debt ratio.

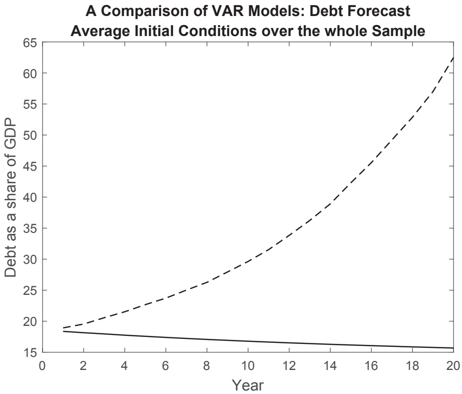
[27] First, debt ratio is included in the estimation of the VAR model in (2). Then the forecasts and impulse responses are calculated based on the macro variables obtained from equation (2) but for each forecasting period debt is not inserted back into equation (2).

Chart 11. **A Comparison of VAR Models: Debt Impulse Responses to Macro Shocks, Average Economic Conditions, GIR Identification**



Notes: The chart shows responses of the debt ratio to one standard deviation positive shocks in primary balance (pb), growth (y), inflation (π), and interest rate (i) under GIR/Cholesky identification. Each chart in the panel presents debt ratio responses from two models: (i) VAR with debt but without debt feedback in computing impulse responses (dashed line); and (ii) VAR with debt feedback (solid line).

Chart 12. **Comparison of VAR Models: Debt Forecast, Starting 2020:Q1**



Note: The chart shows a 5-year forecast of the debt ratio for two different VAR models with average initial economic conditions. The estimation sample is 2000:Q2 – 2019:Q4 (legend).

The debt ratio can rise beyond 50% of GDP in only 5 years (Chart 12) reaching the Maastricht debt rule of 60% of GDP. This unrealistic debt path confirms that the baseline model – a VAR with a debt feedback, produces a robust result where debt dynamics are not explosive, implying that in the long-run, the debt ratio reverts to its stationary level.

Second, the robustness of the results produced by the baseline model is checked in regards to the estimation technique. The baseline model presented in (2) is used to calculate the impulse responses with a different identification method to identify a casual shock in the initial period. As another robustness check of the results produced by the current baseline model, the structural approach of Blanchard and Perotti (2002) is used based on the growing literature applying it. The structural identification scheme is as follows. The relationship between the residuals, u_t, and the structural residuals, ε_t, is: $u_t = B_0\varepsilon_t$, where the residual vector lists variables in the following order: primary deficit, growth, inflation, and interest rate (see Appendix B for details). The B_0 matrix with ones on the diagonal has the following structure: the first row of B_0 matrix includes the elasticity of primary deficit to growth, inflation and interest rate.

Following Mourre, Poissonnier and Lausegger (2019) who present updated calculations and analysis on the semi-elasticities of the budget balance to output for the 28 EU Member States, for the current sensitivity test the value of 0.293 reported by the authors for Bulgaria is used for the semi-elasticity[28] of the

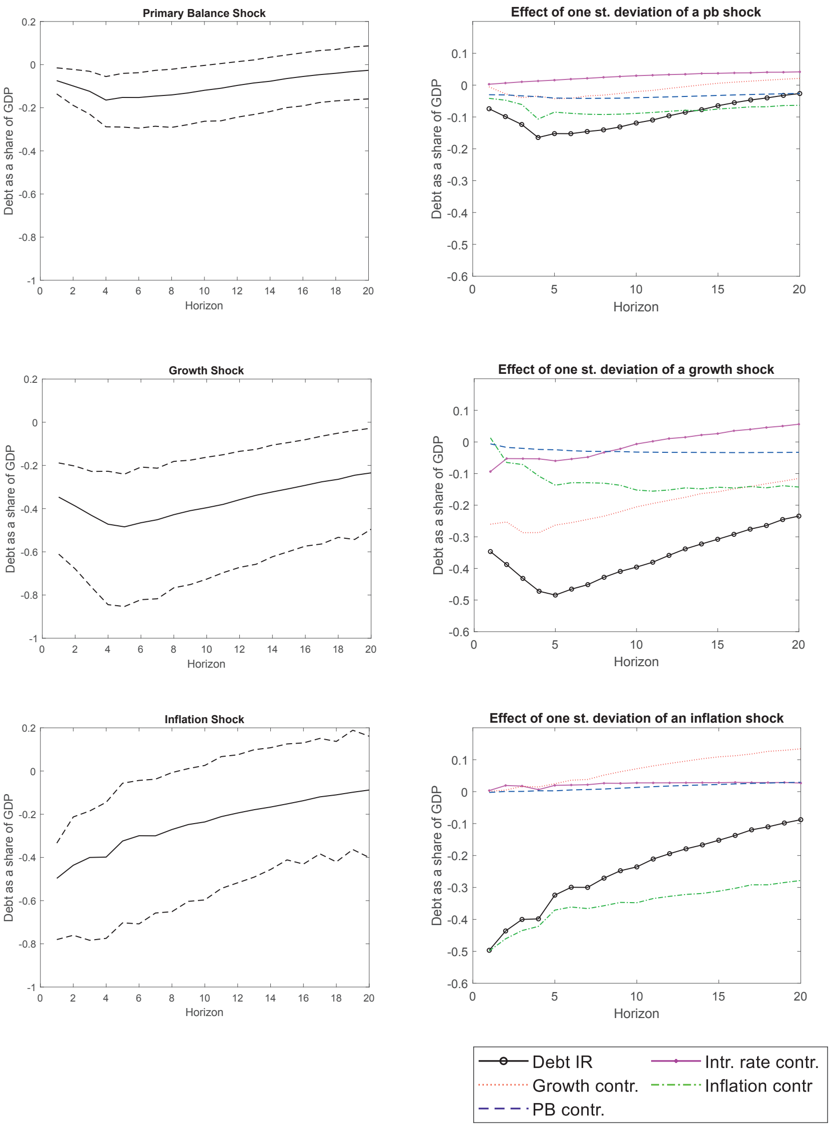
[28] As the current VAR endogenous variables are expressed as percentage of GDP a semi-elasticity is applied, while an elasticity applies to a level (*i.e.* absolute number or monetary amount).

ratio primary (budget) balance/GDP to growth. The value of 0.07 is used for the elasticity of the primary balance ratio to inflation which is in line with the findings presented by Berti, Colesnic, Desponts, Pamies and Sail (2016) from a panel fiscal reaction function for Central and Eastern European countries (iii) zero is assumed for elasticity of the primary balance ratio to interest rate given the low share of interest payments in the total government expenditure[29]. Other parameters in B_0 matrix are identified recursively. The free parameters in B_0 and D matrices (10 remaining elements – see Appendix B) are then estimated using the variance-covariance matrix of the reduced-form residuals. Using the Blanchard-Perotti specification and averaging the initial economic conditions, the magnitude of debt decline is quite close to the one presented in the baseline model under each shock scenario (Chart 13).

Similar to the austerity shock scenario in the main model, the debt decline is statistically significant only in two and a half years following the shock. Again, the main drives in debt decline are the primary balance, positive economic growth and higher inflation, while interest rates counteract the debt reduction. Debt reduction due to an increase in inflation is again statistically significant only in the first few quarters of the horizon. Finally, under Blanchard-Perotti identification positive growth shock can substantially reduce debt where debt decline is statistically significant throughout the whole forecasting horizon. Overall, the main conclusion of the baseline specification remains valid, namely that stimulating growth is more effective in reducing the debt ratio in the medium run.

[29] The share of interest payments in total general government expenditure sharply declines from 10% in 2000 (4.1% of GDP) to its lowest level of 1.5% in 2019 (0.6% of GDP) on the back of the significant debt reduction in the period under consideration and the historically low interest rate environment prevailing in the last decade.

Chart 13. **Debt Impulse Responses to Macro Shocks and Decomposition: Average Conditions, Blanchard-Perotti Identification**



Notes: The chart shows the median responses of the debt ratio and the 95% confidence bands to one standard deviation positive shocks in primary balance (pb), growth (y), inflation (π), and interest rate (i) under GIR identification. The decomposition of the responses to contributions from primary deficit, growth, inflation, and interest rate is also presented. The contributions do not add up exactly due to rounding.

8. Conclusion

This paper studies the dynamics of the Bulgarian debt-to-GDP ratio in response to shocks from major macroeconomic aggregates over the period 2000–2019 using a VAR technique with debt feedback. The results from the current framework suggest that it is important to consider the non-linear relationship between the macroeconomic variables and debt levels. Following an austerity shock and averaging initial economic conditions, in the medium-term, the debt ratio declines but the debt reduction is statistically significant only in the first two and a half years. However, it should be considered that the magnitude of debt response to the shock depends on the initial economic conditions prevailing at the time of the shock. A fiscal consolidation conducted at times of depressed economic conditions and high public debt levels similar to the ones in the period 2001–2003 leads to a more profound decline in the public debt-to-GDP which is statistically significant only in the first year and a half following the shock. However, the positive effect of the austerity shock on public debt is only temporary and fades away in 5 years. The current findings suggest that averaging the economic dynamics of the recent past, an inflation shock, would in fact decrease the debt ratio only for the first three years while the uncertainty around the median debt ratio path is higher in the inflation shock suggesting debt reduction could be more gradual. On the other side, the findings in this study suggest that a positive growth shock on average can substantially reduce debt, where the debt decline response is statically significant in the following 5 years.

Finally, this paper argues that when omitting debt feedback from fiscal VAR models the debt forecast path may not be stable and debt impulse responses could be persistent. Thus, VAR models without debt may not be well suited to study the relationship among macroeconomic variables and debt dynamics. In addition, linear models do not consider initial conditions in regards to impulse responses.

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Appendix A

To determine the number of lags for the debt-to-GDP ratio to be included in (2), the VAR model is simulated out of sample, 1 step ahead, starting from Q1 2016. The preferred number of lags is the one that produces the lowest RMSE. Table 2 below reports the RMSE for different lag lengths. Since the model with 2 lags behaves better than the one with 1 lags and 4 lag, it is selected as a baseline specification.

Table 2. **Out-of-sample RMSE**

	pdef	*g*	*i*	π
RMSE 1 lag	0.025	0.005	0.018	0.003
RMSE 2 lags	0.017	0.004	0.016	0.003
RMSE 4 lags	0.040	0.007	0.020	0.003

Note: The table presents the RMSE obtained simulating the model (2) in a pseudo out-of-sample fashion.
Source: Own calculation.

Appendix B

To describe the GIR methodology and the difference with the OIR function, a brief overview of the standard VAR analysis is presented following Pesaran and Shin (1998):

$$y_t = \sum_{i=1}^{p} \Phi_i \, y_{t-i} + Cw_t + \varepsilon_t, \; t = 1,2,\ldots,T \tag{B.1}$$

where (B.1) represents augmented p^{th}-order Gaussian vector autoregressive model (VAR(p)) with the *mx1* vector of white noise ε_t:

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t, \varepsilon_\tau') = \begin{cases} \Omega & \text{for } t = \tau \\ 0 & \textit{otherwise} \end{cases}$$

With Ω a (mxm) symmetric positive definite matrix. The vector of endogenous variables are collected in $y_t = (y_{1,t}, y_{2,t}, \ldots, y_{m,t})'$, w_t is a ($qx1$) vector of deterministic and/or exogenous variables and Φ_i, $i = 1, 2, \ldots$, p, and C are respectively *mxm* and *mxq* coefficient matrices. If each side of (B.1) is premultiplied by B_0, the result is

$$B_0 y_t = \sum_{i=1}^{p} B_0 \Phi_i \, y_{t-i} + B_0 C w_t + B_0 \varepsilon_t \tag{B.2}$$

where

$$u_t = B_0 \varepsilon_t \tag{B.3}$$

$$\varepsilon_t = B_0^{-1} u_t \tag{B.4}$$

represent the relationship between the structural shocks, u_t, and the VAR innovations ε_t. B_0 is the lower triangular matrix containing the structural parameters (or short-run elasticities) with unit coefficients along the principal diagonal. Equation (B.2) is recognized as the vector autoregressive representation for the dynamic structural vars (SVAR). The identification conditions *a là* Sims (1980) is given by:

$$Cov(u_t) = E(u_t, u_\tau') = D \tag{B.5}$$

where D is a diagonal matrix whose (j,j) element is the variance of u_{jt}

$$Cov(\varepsilon_t) = E(\varepsilon_t, \varepsilon_\tau') = \Omega = B_0^{-1} E(u_t u_t')(B_0^{-1})' = B_0^{-1} D (B_0^{-1})' \tag{B.6}$$

Letting $D^{1/2}$ denote the diagonal matrix whose (j,j) element is the standard deviation of u_t, the above could be rewritten as

$$\Omega = B_0^{-1}D(B_0^{-1})' = B_0^{-1}D^{1/2}D^{1/2}(B_0^{-1})' = PP' \quad \text{(B.7)},$$

where the expression $P = B_0^{-1}D^{1/2}$ is the Cholesky decomposition of the matrix Ω. Like B_0^{-1}, the (mxm) matrix P is lower triangular, though whereas B_0^{-1} has 1s along its principal diagonal, P has standard deviation of u_t along its principal diagonal.

Thus, a VAR as described in (B.1) can be viewed as the reduced form of a general dynamic structural model. Under the assumption of covariance stationarity, the Wold theorem implies that it is possible to 'invert' (B.1), that is, it is possible to reformulate the model as an infinite order moving average (MA(∞))[30]:

$$y_t = \sum_{i=0}^{\infty} \Psi_i\, \varepsilon_{t-i} + \sum_{i=0}^{\infty} G_i\, w_{t-i},\ t = 1,2,\ldots,T \quad \text{(B.8)}$$

with the moving average mxm matrix Ψ_i that can be obtained using the following recursive relations:

$$\Psi(L) = [\Phi(L)]^{-1} \quad \text{(B.9)}$$

and in general, for L^s:

$$\Psi_i = \Phi_1\Psi_{i-1} + \Phi_2\Psi_{i-2} + \ldots + \Phi_p\Psi_{p-1},\ i = 1,2,\ldots \quad \text{(B.10)}$$

with $\Psi_0 = I_m$ and $\Psi_i = 0$ for $i < 0$, and $G_i = \Psi_i g$.

The sequence Ψ_0, Ψ_1... actually represents the impulse response function for model (B.2). This is clearly presented by moving (B.2) forward by h periods to obtain:

$$y_{t+h} = \sum_{i=0}^{\infty} G_i\, w_{t+h} + \Psi_0\varepsilon_{t+h} + \Psi_1\varepsilon_{t+h-1} + \ldots + \Psi_{h-1}\varepsilon_{t+1} + \Psi_h\varepsilon_t + \Psi_{h+1}\varepsilon_{t-1}\ldots \quad \text{(B.11)}.$$

The traditional orthogonalized impulse response function (OIR) is designed to provide an answer to the question: "What is the effect of a shock of size ϑ hitting the system at time t on the state of the system at time $t + m$, given that no other shocks hit the system?". The OIR function is based on decomposing the original var innovations $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)$ into a set of uncorrelated components $(u_1, u_2, \ldots, u_m)$ and calculating the consequences for y_{t+h} for unit impulse in u_{jt}, *i.e.* purely exogenous and independent structural shock. The OIR approach requires the researcher to take a stand on how to recover and/or identify structural shocks which in general are not directly observable. Different approaches are used in the empirical work in order to recover the structural shocks. The traditional approach, suggested by Sims (1980), is to identify the exogenous

[30] For more details, see Pesaran and Shin, 1998, Generalized impulse response analysis in linear multivariate models, Economics Letters, 58(1): 17–29.

shocks by using Cholesky decomposition of the matrix Ω (21) where P is an mxm is a lower triangular matrix. Thus, (B.8) could be rewritten as:

$$y_t = \sum_{i=0}^{\infty}(\Psi_i P)(P^{-1}\varepsilon_{t-i}) + \sum_{i=0}^{\infty} G_i w_{t-i} = \sum_{i=0}^{\infty}(\Psi_i P)u_{t-i} + \sum_{i=0}^{\infty} G_i w_{t-i}, \quad t = 1,2,..T \qquad (B.12)$$

where $u_{t-i} = P^{-1}\varepsilon_t$ are orthogonalized, namely $E(u_t, u'_t) = I_m$. Hence, the $mx1$ vector of the orthogonalized impulse response (OIR) function of a unit shock to the j^{th} element on y_{t+h} is given by:

$$\psi_j^o(h) = \Psi_h P e_j, \quad h = 0,1,2,.. \qquad (B.13),$$

where e_j is an $mx1$ vector with unity as its j^{th} element and zeros elsewhere[31].

An alternative approach to the OIR would be the GIR (12) but instead of shocking all elements in the vector ε_t, only one element would be chosen to be shocked, j^{th} element, integrating out the effects of other contemporaneous and future shocks using an assumed or the historical observed distribution of the errors. Assuming that ε_t is normally distributed[32] and given that $E(\varepsilon_t, \varepsilon'_t) = \Omega$ for all t, where $\Omega = \{\sigma_{ij}, i,j = 1,2,...m\}$ Koop, Pesaran and Potter (1996) show that:

$$E\left(\varepsilon_t \middle| \varepsilon_{jt} = \vartheta_j\right) = (\sigma_{1j}, \sigma_{2j}, \sigma_{3j}, \ldots \sigma_{mj})' \sigma_{jj}^{-1}\vartheta_j = \Omega e_j \sigma_{jj}^{-1}\vartheta_j \qquad (B.14).$$

The $mx1$ vector of unscaled generalized impulse response of the effect of a shock to the j^{th} equation at time t on y_{t+h} is given by:

$$\left(\frac{\Psi_h \Omega e_j}{\sqrt{\sigma_{jj}}}\right)\left(\frac{\vartheta_j}{\sqrt{\sigma_{jj}}}\right), \quad h = 0,1,2, \ldots \qquad (B.15).$$

And by setting $\vartheta_j = \sqrt{\sigma_{jj}}$ the scaled generalized impulse response function is obtained:

$$\psi_j^g(h) = \sigma_{jj}^{-1/2}\Psi_h \Omega e_j, \qquad h = 0,1,2, \ldots \qquad (B.16),$$

which measures the effect of one standard error shock to the j^{th} equation at time t on expected values of y at time $t+h$.

[31] By the 'unit' shock, it is meant a shock with size equal to one standard deviation.

[32] When the distribution of the errors ε_t are non-normal, Pesaran and Shin (1998) propose to obtain the conditional expectations $E\left(\varepsilon_t \middle| \varepsilon_{jt} = \vartheta_j\right)$ by stochastic simulations or by resampling techniques if the distribution of errors is not known.

Appendix C

This appendix follows closely Granger and Teräsvirta (1993, Ch. 8). The authors give a detailed description of various methods of computing the conditional expectation of a nonlinear simple bivariate model:

$$y_t = g(x_{t-1}) + \varepsilon_t \tag{C.1}$$

for example

$$y_t = x_{t-1}^2 + \varepsilon_t \tag{C.2}$$

where x_t is AR(1) process and ε_t has a zero mean, independent and identically distributed.

Using the least square criterion, the optimal one step forecast is

$$f_{t,1}^{y} = E[y_{t+1}|I_t] = g(x_t), \text{ where } E[e_{t+1}|I_t] = 0 \qquad \text{(C.3),}$$

where I_t: x_{t-i}, y_{t-i} is the information set available at time t. Thus, if one knows g(), or has an acceptable approximation for it, one-step forecasts can be achieved without difficulties.

The two-step forecast is not as easy. The optimum two-two step forecast is:

$$f_{t,2}^{y} = E[y_{t+2}|I_t] = E[g(x_{t+1})|I_t] \qquad \text{(C.4).}$$

As x_{t+1} is not usually known at time t, it is necessary to specify the generating mechanism for x_t. A good approximation to this mechanism is available and for ease of illustration, let this be AR(1) model:

$$x_t = \alpha x_{t-1} + e_t \qquad \text{(C.5).}$$

Where e_t is i.i.d. with zero mean and distribution D. This gives one-step OLS forecast:

$$f_{t,1}^{y} = \alpha x_t \qquad \text{(C.6).}$$

Four alternative two-steps forecasts in the form:

$$f_{t,2}^{y} = E\left[g(f_{t,1}^{x} + e_{t+1}|I_t\right] \tag{C.7}$$

are:

1) naive: $fn^{y}_{t,2} = g(f^{x}_{t,1})$,

ignoring the process e_{t+1} by putting its value to zero;

2) exact: $fe^{y}_{t,2} = \int_{-\infty}^{\infty} g(f^{x}_{t,1} + z)d\Phi\,(z)$,

where $\Phi(z)$ is the distribution function of *D*. Thus, if e_t is $N(0, \sigma^2)$ then $\Phi(z)$ is the normal distribution function;

3) *Monte Carlo:* $fm^{y}_{t,2} = \frac{1}{N}\sum_{j=1}^{N} g(f^{x}_{t,1} + z_j)$,

where z_j, $j = 1, 2,.. N$ are random numbers drawn from the distribution *D*. For *N* large enough *fm* and *fe* should be identical;

4) *bootstrap* $fb^{y}_{t,2} = \frac{1}{n-1}\sum_{j=1}^{n-1} g(f^{x}_{t,1} + e_j)$,

where e_j, $j = 1, 2... n-1$ are the $n–1$ values of the residual e_t observed over the sample period.

For the particular model the forecasts will be as follow:

$$fn^{y}_{t,2} = \alpha^2 x_t^2$$

$$fe^{y}_{t,2} = fm^{y}_{t,2} = \alpha^2 x_t^2 + \sigma_e^2$$

$$fb^{y}_{t,2} = \alpha^2 x_t^2 + \hat{\sigma}_e^2$$

Appendix D

Following Cherif and Hasanov (2012) the decomposition of the debt impulse response, d^{IR}, is defined in terms of the contribution of each macroeconomic aggregate as follows:

$$d_t^{IR} = d_t^s - d_t^n = pb_t^* + i_t^* - \pi_t^* - g_t^*,$$

where *s* and *n* stand for 'shock' and 'no shock' debt paths. Using debt dynamics equation (3) in the text and approximating the nonlinear component, the components of the decomposition at time *t* are:

$$pb_t^* = (pb_t^s - pb_t^n) + (1 + i_t^s - \pi_t^s - g_t^s)pb_{t-1}^*$$

$$i_t^* = (i_t^s - i_t^n)d_{t-1}^n + (1 + i_t^s - \pi_t^s - g_t^s)i_{t-1}^*$$

$$\pi_t^* = (\pi_t^s - \pi_t^n)d_{t-1}^n + (1 + i_t^s - \pi_t^s - g_t^s)\pi_{t-1}^*$$

$$g_t^* = (g_t^s - g_t^n)d_{t-1}^n + (1 + i_t^s - \pi_t^s - g_t^s)g_{t-1}^*$$

The first part of the right hand side of the equation is the difference between 'shock' and 'no shock' paths of the components scaled by the previous 'no shock' debt ratio. The second term is the adjusted previous value of the component. Thus, the debt impulse response decomposition is:

$$d_t^{IR} = \Delta^{s/n} pb_t + (\Delta^{\frac{s}{n}} i_t - \Delta^{\frac{s}{n}} \pi_t - \Delta^{\frac{s}{n}} g_t)d_{t-1}^n + (1 + i_t^s - \pi_t^s - g_t^s)d_{t-1}^{IR},$$

where $\Delta^{s/n}$ stands for the difference between 'shock' and 'no shock' paths.

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